Mathematics Core Academic Standards Impact and Implications

Cindy Bryant
Mathematics Consultant
Cindy.Bryant@dese.mo.gov

June 27, 2012
# Traditional Approach to U.S. Mathematics Curriculum

<table>
<thead>
<tr>
<th>GRADES</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9 - 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Number and Operations
- Algebra Relationships
- Geometry
- Measurement
- Data Analysis and Probability

“curriculum that is “a mile wide and an inch deep.”” CCSS, 2
How does the structure of the CCSS (MO Mathematics CAS) impact the teaching and learning of mathematics?
• The standards are meant to be a blueprint for math instruction that is more focused and coherent.

• The focus and coherence in this blueprint is largely in the way the standards progress from each other, coordinate with each other and most importantly cluster together into coherent bodies of knowledge.

• Crosswalks and alignments and pacing plans and such cannot be allowed to throw away the focus and coherence and regress to the mile-wide curriculum.
Traditional Approach to U.S. Mathematics Curriculum

<table>
<thead>
<tr>
<th>GRADES</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9 - 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and Operations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Algebra Relationships</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data Analysis and Probability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

“curriculum that is “a mile wide and an inch deep.” CCSS, 2

Missouri Core Academic Standards in Mathematics
Domains and Conceptual Categories K - 12

<table>
<thead>
<tr>
<th>GRADES</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9 - 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement and Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Functions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Functions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Functions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Functions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number and Operations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fractions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratios &amp; Proportional Relationships</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number and Operations in Base Ten</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Number System</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operations and Algebraic Thinking</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expressions and Equations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Algebra</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

• Time in each grade to focus on what matters
• Time to finish what you begin

MODELING
Why MO Core Academic Standards

• Common Core State Standards (CCSS) initiative only included English language arts and mathematics contents
• Common Next Generation Science Standards will soon be available for adoption
• Work is being done on common social studies standards

• Core Academic Standards (CAS) can be used for all MO academic standards not just those that were part of the CCSS initiative
Missouri Core Academic Standards in Mathematics

**Domains and Conceptual Categories K - 12**

<table>
<thead>
<tr>
<th>GRADES</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9 - 12</th>
</tr>
</thead>
</table>

- **Measurement and Data**
  - Counting and Cardinality

- **Number and Operations in Base Ten**

- **Operations and Algebraic Thinking**

- **Number and Operations Fractions**

- **Ratios & Proportional Relationships**

- **The Number System**

- **Expressions and Equations**

- **Geometry**

- **Functions**

- **Number and Quantity**

- **Algebra**

- **Statistics and Probability**

- **Statistics and Probability**

- **MODELING**

• Time in each grade to focus on what matters
• Time to finish what you begin
High School Mathematics Content Standards

• All CCSS coded with (+) identify content that would be included in advanced courses.

• All CCSS coded with (★) indicate modeling standards.
When you read a book...

- Introduction
- Standards for Mathematical Practice K - 12
- Mathematics Content Standards K – 12
- Glossary
- Sample of Works Consulted
College and Career Ready

Common Core State Standards
Mathematics

Content Standards
(Show-Me Content Standards)

Procedure & Understanding
Common Language

- “These Standards define what students should understand and be able to do in their study of mathematics.” (Common Core State Standards, 2010)

- “These standards (73 in all) are intended to define what students should learn by the time they graduate from high school.” (The Show-Me Standards, 1996)

- “However, these Standards do provide clear signposts along the way to the goal of college and career readiness for all students.” (Common Core State Standards, 2010)

- “However, graduates who meet these standards should be well-prepared for further education, work, and civic responsibilities.” (The Show-Me Standards, 1996)
## Critical Areas of Instruction

### Kindergarten
In Kindergarten, Instructional time should focus on two critical areas:
1. Representing and comparing whole numbers, initially with sets of objects; and
2. Describing shapes and space.

More learning time in Kindergarten should be devoted to number than to other topics.

### Grade 1
In Grade 1, Instructional time should focus on four critical areas:
1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; and
2. Developing understanding of whole number relationships and place value, including grouping in tens and ones; and
3. Developing understanding of linear measurement and measuring lengths in iterating units of length; and
4. Reasoning about attributes of, and composing and decomposing geometric shapes.

### Grade 2
In Grade 2, Instructional time should focus on four critical areas:
1. Extending understanding of base ten notation; and
2. Building fluency with addition and subtraction; and
3. Using standard units of measure; and
4. Describing and analyzing shapes.

### Grade 3
In Grade 3, Instructional time should focus on four critical areas:
1. Developing understanding of multiplication and division and strategies for multiplication and division with 100; and
2. Developing understanding of fractions, especially unit fractions (fractions with numerator 1); and
3. Developing understanding of the structure of rectangular arrays and of area; and
4. Describing and analyzing two-dimensional shapes.

### Grade 4
In Grade 4, Instructional time should focus on three critical areas:
1. Developing understanding and fluency with multi-digit multiplication and, and developing understanding of dividing to find quotients involving multi-digit dividends; and
2. Developing understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and
3. Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measurements, and symmetry.

### Grade 5
In Grade 5, Instructional time should focus on three critical areas:
1. Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); and
2. Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and
3. Developing understanding of volume.

### Grade 6
In Grade 6, Instructional time should focus on four critical areas:
1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; and
2. Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; and
3. Writing, interpreting, and using expressions and equations; and
4. Developing understanding of statistical thinking.

### Grade 7
In Grade 7, Instructional time should focus on four critical areas:
1. Developing understanding of and applying proportional relationships; and
2. Developing understanding of operations with rational numbers and working with expressions and linear equations; and
3. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and
4. Drawing inferences about populations based on samples.

### Grade 8
In Grade 8, Instructional time should focus on three critical areas:
1. Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; and
2. Grasping the concept of a function and using functions to describe quantitative relationships; and
3. Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.
One of the characteristics of the most effective schools is their willingness to declare that some things are more important than others; they are willing to abandon some less important content so as to be able to have enough time dedicated to those areas that are valued most.

High School Critical Areas of Focus

Appendix A: Designing High School Mathematics Courses

http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf
Conceptually Engaging Tasks are Atypical

Typical classroom mathematics teaching in the United States tends not to use challenging tasks, nor to promote students’ thinking about and engagement with mathematical ideas, and thus fails to help students develop understanding of the mathematics they are learning.

Conceptual Understanding

The CAS progressions for the different content domains:

1. Develop conceptual understanding often building on children’s informal knowledge
2. Support conceptual knowledge and develop informal strategies to solve problems within the domain
3. Refine the informal strategies to develop fluency with standard procedures

Much more about conceptual understanding in July...
Grade 5 Overview

Operations and Algebraic Thinking
- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten
- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions
- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

The Number System
- Gain familiarity with concepts of positive and negative integers.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practice Standards are included throughout K – 12. Implementation can begin immediately.
Missouri Core Academic Standards in Mathematics

Domains and Conceptual Categories K - 12

<table>
<thead>
<tr>
<th>GRADES</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9 - 12</th>
</tr>
</thead>
</table>

STANDARDS FOR MATHEMATICAL PRACTICE

- Measurement and Data
- Statistics and Probability
- Statistics and Probability
- Counting and Cardinality
- Number and Operations Fractions
- Ratios & Proportional Relationships
- Functions
- Number and Quantity
- The Number System
- Expressions and Equations
- Algebra
- Geometry

- Time in each grade to focus on what matters
- Time to finish what you begin

MODELING
Standards of Mathematical Practice

- Processes
  - *NCTM, PSSM*
- Proficiencies
  - *NRC, Adding It Up*
Standards for Mathematical Practice

Aligned to the Missouri Show-Me Standards
Proficiencies

- **Adaptive Reasoning** – capacity for logical thought, reflection, explanation, and justification
- **Strategic Competence** – ability to formulate, represent, and solve mathematical problems
- **Conceptual Understanding** – comprehension of mathematical concepts, operations, and relations
- **Procedural Fluency** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Productive Disposition** – habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one’s own efficacy.
Standards for Mathematical Practice

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Reasoning and Explaining

Modeling and Using Tools

Seeing Structure and Generalizing
# The Standards for Mathematical Practice

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(3.1 – 3.7)</td>
<td>(1.7, 1.10, 3.5, 3.8)</td>
<td>(1.7,1.8, 3.3, 3.5)</td>
<td>(1.6,1.8, 1.10,2.1 3.3 4.1)</td>
</tr>
<tr>
<td>2.</td>
<td>Reason abstractly and quantitatively.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Construct viable arguments and critique the reasoning of others.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Model with mathematics.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Us appropriate tools strategically.</td>
<td>Attend to precision.</td>
<td>Look for and make use of structure.</td>
<td>Look for and express regularity in repeated reasoning.</td>
</tr>
<tr>
<td>6.</td>
<td>(1.4, 1.10, 2.7)</td>
<td>(1.7, 2.2, 2.3, 3.8)</td>
<td>(1.6, 1.7, 1.8, 2.3, 3.1, 3.6)</td>
<td>(1.6, 3.5, 3.6, 3.7)</td>
</tr>
<tr>
<td>7.</td>
<td>Look for and make use of structure.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Look for and express regularity in repeated reasoning.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Missouri Show-Me Process Standards*
Grades K – 5 Specific Shifts

Introduction at earlier grade

Addition, subtraction, multiplication, and division of whole numbers and other rational numbers.

Introduction at later grade

Introduced as a domain in Grade 6:
• Statistics & Probability
• Expressions & Equations
Missouri Core Academic Standards in Mathematics
Domains and Conceptual Categories K - 12

**STANDARDS FOR MATHEMATICAL PRACTICE**

- Measurement and Data
- Statistics and Probability
- Statistics and Probability
- Functions
- Number and Quantity
- Algebra
- Geometry
- Expressions and Equations
- The Number System
- Ratios & Proportional Relationships
- Number and Operations
- Fractions
- Number and Operations in Base Ten
- Operations and Algebraic Thinking
- Counting and Cardinality

**GRADES**

K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 - 12

• Time in each grade to focus on what matters
• Time to finish what you begin
Focus on operations with whole numbers, fractions and decimals to provide the foundation for the more demanding mathematical concepts and procedures experienced in grades 6 – 12.
High School Specific Shifts

More specificity

• Modeling★
• Statistics and probability
• Geometry proofs
Fraction Shifts

“It is possible to have good number sense for whole numbers, but not for fractions.”

Sowder, J. and Schappelle, Eds. 1989
2. A major goal of K – 8 mathematics education should be proficiency with fractions (including decimals, percent, and negative fractions), for such proficiency is foundational for algebra and seems to be severely underdeveloped. In addition, the Panel identified Critical Foundations of Algebra (p 17).
Problem: \( \frac{7}{8} - \frac{1}{8} = ? \)

**Interviewer:** Melanie these two circles represent pies that were each cut into eight pieces for a party. This pie on the left had seven pieces eaten from it. How much pie is left there? **Melanie:** One-eighth, writes \( \frac{1}{8} \)

**Interviewer:** The pie on the right had three pieces eaten from it. How much is left of that pie? **Melanie:** Five-eighths, writes \( \frac{5}{8} \)

**Interviewer:** If you put those two together, how much of a pie is left? **Melanie:** Six-eighths, writes \( \frac{6}{8} \).

**Interviewer:** Could you write a number sentence to show what you just did? **Melanie:** Writes \( \frac{1}{8} + \frac{5}{8} = \frac{6}{16} \).

**Interviewer:** That’s not the same as you told me before. Is that OK? **Melanie:** Yes, this is the answer you get when you add.
Fractions

Facets of the lack of student conceptual understanding:

• Not viewing fractions as numbers at all, but rather as meaningless symbols that need to be manipulated in arbitrary ways to produce answers that satisfy a teacher.
• Focusing on numerators and denominators as separate numbers rather than thinking of the fraction as a single number.
• Confusing properties of fractions with those of whole numbers.
• Grade 1
Partition and describe two and four equal shares of circles and rectangles ...recognize that decomposing into more equal shares created similar figures

Grade 2
• Partition and describe two, three or four equal shares of circles and rectangles, ...recognize that equal shares of identical wholes need not have the same shape
CAS Fractions

Grade 3
• Developing understanding of fractions, especially unit fractions

Grade 4
• Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers
Grade 5
• Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)

Grade 6
• Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems;
• Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers
Build fraction understanding from whole number understanding.

The number line

0 1 2 3 4 5 6 etc.

The number line marked off in thirds

0 1 2 3 4 5 6
0 1 2 3 4 5 6 7 8 9 10 11 12
Build fraction understanding from whole number understanding.

Number line representation of \( \frac{5}{3} \)

One part of a division of the unit interval into 3 parts of equal length

5 parts

the point \( \frac{5}{3} \) on the number line
Build fraction understanding from whole number understanding.

Fraction equivalence on the number line.

\[
\frac{4}{3} = \frac{5 \times 4}{5 \times 3}
\]
## Algebra in elementary school is not about patterns

<table>
<thead>
<tr>
<th>Grade</th>
<th>Operations and Algebraic Thinking</th>
<th>Number and Operations in Base Ten</th>
<th>Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Understand and apply properties of operations and the relationships between addition and subtraction.</td>
<td>Use place value understanding and properties of operations to add and subtract.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Use place value understanding and properties of operations to add and subtract.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Understand properties of multiplication and the relationship between multiplication and division.</td>
<td>Use place value understanding and properties of operations to perform multi-digit arithmetic.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Use place value understanding and properties of operations to perform multi-digit arithmetic.</td>
<td></td>
<td>Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.</td>
</tr>
<tr>
<td>5</td>
<td>Use place value understanding and properties of operations to perform multi-digit arithmetic.</td>
<td></td>
<td>Apply and extend previous understandings of multiplication and divisions to multiply and divide fractions.</td>
</tr>
</tbody>
</table>
# Expressions and Equations

## Grade 6
- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

## Grade 7
- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

## Grade 8
- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.
Focus in high school:
unifying isolated techniques

<table>
<thead>
<tr>
<th>Algebra</th>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Seeing Structure in Expressions</td>
<td>• Interpreting Functions</td>
</tr>
<tr>
<td>• Arithmetic with Polynomials and Rational Expressions</td>
<td>• Building Functions</td>
</tr>
<tr>
<td>• Creating Equations</td>
<td>• Linear, Quadratic, and Exponential Models</td>
</tr>
<tr>
<td>• Reasoning with Equations and Inequalities</td>
<td>• Trigonometric Functions</td>
</tr>
</tbody>
</table>
# Elements of High Quality K – 12 Mathematics Classrooms

| S = student learning | T = teacher instruction | **Instruction and Learning Elements in K – 12 Mathematics Classrooms** | **MP** = Mathematical Practices  
| **SP** = Show-Me Process Standards |
|----------------------|-------------------------|---------------------------------------------------------------------|-------------------------------------------------------------------|
| **A.** Using questioning techniques to facilitate learning | | **MP** = 3, 6  
| **SP** = 1.4, 1.7, 1.8, 2.2, 2.3, 3.3, 3.5, 3.8 |
| **B.** Actively engaging in the learning process | | **MP** = 1  
| **SP** = 3.1 – 3.7 |
| **C.** Choosing “good” problems – ones that invite exploration of an important mathematical concept and allow the chance to solidify and extend knowledge | | **MP** = 1, 4, 7, 8  
| **SP** = 1.6, 1.7, 1.8, 1.10, 2.1, 2.3, 3.1 – 3.7 |
| **D.** Using existing mathematical knowledge to make sense of the task | | **MP** = 1, 2, 3  
| **SP** = 1.7, 1.8, 1.10, 3.1 – 3.8 |
| **E.** Making connections among mathematical concepts | | **MP** = 2, 7, 8  
| **SP** = 1.6 – 1.8, 1.10, 2.3, 3.1, 3.5 – 3.8 |

*Administrator’s Guide: Interpreting the Common Core State Standards to Improve Mathematics Education (NCTM, 2010)*
### Instruction and Learning Elements in K – 12 Mathematics Classrooms

<table>
<thead>
<tr>
<th>Instruction Teacher Strategies</th>
<th>Learning Student Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>C. Choosing “good” problems – ones that invite exploration of an important mathematical concept and allow students the chance to solidify and extend their knowledge</td>
<td>B. Actively engaging in the learning process</td>
</tr>
<tr>
<td>F. Assessing students’ understanding by listening to discussions and asking students to justify their responses</td>
<td>D. Using existing mathematical knowledge to make sense of the task</td>
</tr>
<tr>
<td>A. Using questioning techniques to facilitate learning</td>
<td>E. Making connections among mathematical concepts</td>
</tr>
<tr>
<td>I. Encouraging students to explore multiple solutions</td>
<td>G. Reasoning and making conjectures about the problem</td>
</tr>
<tr>
<td>N. Challenging students to think more deeply about the problems they are solving and to make connections with other ideas within mathematics</td>
<td>H. Communicating their mathematical thinking orally and in writing</td>
</tr>
<tr>
<td>P. Creating a variety of opportunities, such as group work and class discussions, for students to communicate mathematically</td>
<td>K. Listening and reacting to others’ thinking and solutions to problems</td>
</tr>
<tr>
<td>J. Modeling appropriate mathematical language and a disposition for solving challenging mathematical problems</td>
<td>L. Using a variety of representations, such as pictures, tables, graphs, and words, for their mathematical thinking</td>
</tr>
<tr>
<td></td>
<td>M. Using mathematical and technological tools, such as physical materials, calculators, and computers, along with textbooks and other instructional materials</td>
</tr>
<tr>
<td></td>
<td>O. Building new mathematical knowledge through problem solving</td>
</tr>
</tbody>
</table>

*Administrator’s Guide: Interpreting the Common Core State Standards to Improve Mathematics Education (NCTM, 2010)*
Fostering Engagement

• The teacher creates a climate that supports mathematical thinking and communication (MP 2 and 3).
• Students are accustomed to explaining their ideas and questioning solutions that don’t make sense to them (MP 3).
• Students are not afraid to take risks and know that it is acceptable to struggle with some ideas and to make mistakes (MP 1)

Fostering Engagement

• The teacher responds in a way that keeps the focus on thinking and reasoning rather than only getting the right answer (MP2).

• Incorrect answers and ideas are not simply judged wrong – the teacher helps identify parts of student thinking that may be correct, sometimes leading students to a new idea and solutions that are correct.
Resources

• Developing Effective Fraction Instruction for Kindergarten Through 8th Grade
• Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools
• Common Core State Standards Progressions
  http://ime.math.arizona.edu/progressions/ or http://commoncoretools.me/
• Illustrative Mathematics Project
  http://illustrativemathematics.org/
• National Council of Teachers of Mathematics
  www.nctm.org