



Teacher's Guide: Early Mathematics

Purpose

This guide, along with the *Parent Handbook*, is a companion to the *Missouri Pre-K Mathematics Standards*. It is intended for all adults who work with preschool-age children and their families — teachers, caregivers and/or parent educators. It explains the standards and contains practical suggestions for creating engaging, developmentally appropriate learning communities to foster each child’s mathematical thinking.

Why is it important to have mathematics standards for pre-kindergarten learners? In Missouri, the Outstanding Schools Act of 1993 called together a group of master teachers, parents and policy-makers from around the state. The result of that group’s work are the Show-Me Standards, which are designed for students in kindergarten through grade 12 and serve to ensure that graduates of Missouri’s public schools have the knowledge, skills and competencies to lead productive, fulfilling and successful lives. However, we all realize the foundations for learning are laid well before a child enters kindergarten! Research tells us the pre-kindergarten years provide crucial opportunities for educators and caregivers to influence children’s growth and development. If our goal is that every child enter kindergarten ready to learn and succeed, pre-kindergarten educational standards can provide us with shared understandings about the competencies critical for this to occur. In addition, pre-kindergarten standards provide a direct link to the Show-Me Standards, so we know that we are preparing children for the high expectations they will encounter as they progress through school.

To this end, the *Missouri Pre-K Mathematics Standards* describe what most children should know and be able to do in the area of mathematics by the time they enter kindergarten. They represent a shared set of expectations for preschool children, expectations developed by drawing upon current research about how young children learn. It is important to keep in mind, however, that children learn and develop in their own unique ways. While research demonstrates that these standards are appropriate for most children who are about to enter kindergarten, our responsibility as educators is to assess where each child is on the continuum of mathematical thinking and build on what that child knows and can do.

Ongoing observational assessment is a key element in supporting children’s early mathematical development. This guide will illustrate some indicators that teachers can reliably assess to show each child’s progress over time. This data can be used to plan meaningful, engaging learning experiences that promote both mathematical development and a love for solving problems. In addition, you will find information about creating mathematically rich environments and partnering with parents and families to promote mathematical thinking and an enthusiasm for learning. Lists defining the mathematical terms referred to in the standards and outlining helpful resources are also included.

Guiding Principles

Missouri early childhood practices are based on the following principles. They provide a structure to support our work with young children and remind us of the “big picture” — the theoretical framework for our teaching.

- 1. All children actively seek to comprehend the world in which they live.** Given the opportunity to make choices concerning their activities, they acquire knowledge, skills and the ability to solve problems. Children are born with a desire to learn about and make sense of their world. Research has shown us that if children are able to choose what they would like to find out about, they not only gain knowledge and skills but are highly motivated to do so. We can foster this early love for learning by allowing children choices in their activities and by supporting their attempts to solve their own problems.
- 2. Children construct knowledge and values through interactions with peers, parents and other adults and through active exploration of the physical and social environments.** Children flourish in a learning community where they can directly act on objects and interact with people. This is because, for young children, thoughts and actions are very closely related. When we provide children with opportunities to explore, experiment, make predictions, collaborate and share their thinking with others, we support both their cognitive and social development.
- 3. Young children’s thinking contains predictable errors.** As children develop, they construct knowledge by integrating new information with what they already know. In doing so, children will often make errors or mistaken assumptions. This is a necessary part of the learning process. When we give children enough time and appropriate guidance to recognize and correct their errors, we not only teach them how to think for themselves, but we show them that we have confidence that they can figure things out. Children with confidence in their own ability to work through problems are active thinkers!
- 4. Early learning and areas of development interact and influence each other.** While adults are accustomed to categorizing learning by subject areas (science, math, etc.), this is not how young children organize their thoughts. Their emotional and social development goes hand in hand with their learning in other areas. In fact, a child’s ability to build a knowledge base depends upon his or her social, emotional and physical development and is closely linked to it. As educators, we serve children best by designing learning experiences that are both meaningful to them and that span various areas of development.

- 5. Families (parents) are the child’s first and most important teachers.** Families (parents), as children’s earliest and most influential teachers, are our most helpful partners in educating young children. We are all deeply invested in the child’s success and share a common goal: to provide the best education possible in a safe, nurturing environment that is rich with opportunities for learning. Building open, respectful and trusting relationships with the parents and families of the children in our care helps children develop a sense of security and continuity between home and school.
- 6. Children exhibit individual differences in their development of competencies.** Although research (and our own experience) has shown us that children generally go through identifiable stages as they grow and develop, it is also true that there can be great individual differences in the rate and manner in which children pass through these stages. This variation is normal. We can best support each child’s progress by meeting children where they are and building on their strengths.

children flourish in a learning community where they can directly act on objects and interact with people

Organization

The standards cover the broad scope of child development and are organized by:

- I. Content Component** — the specific content area to be addressed (e.g., number and operations or measurement).
- II. Process Standards** — identifiable competencies or capabilities in the process of mathematical development, such as solving problems using number or using numerical representation.
- III. Indicators** — observable milestones in the development of competencies, such as writing some numerals or taking objects apart and putting them together again.
- IV. Examples** — specific behaviors children may exhibit in their mathematical development.

This structure provides us with an accessible way to see how the standards fit into our curriculum and teaching practices. Definitions of the terms and concepts used in the standards are listed on pages 6 through 8.

Mathematics Standards (Goals) for the Preschool Years

Below are the Standards (Goals) for Pre-K Mathematics as identified by the Missouri Department of Elementary and Secondary Education, Early Childhood Section.

Number and operations

- Uses number to show quantity.
- Uses language to represent number of objects.
- Solves problems using number.
- Uses numerical representation.

Geometry and spatial sense

- Investigates positions and locations.
- Explores shapes in the environment.

Patterns and relationships

- Recognizes relationships in the environment.
- Uses patterns in the environment.

Measurement

- Makes comparisons.
- Uses measurement.

Exploring data

- Collects, organizes and uses information.

Mathematical Concepts Defined

Content component: Number and operations

Number and operations: understanding of numbers, ways of representing numbers, relationships among numbers and number systems

Number: a unit belonging to a mathematical system used for counting, measuring, ordering and labeling; the meaning of a number word or numeral

Number sense: the ability to understand numbers, ways of representing numbers and relationships among numbers (Number sense is much more than counting; it involves the ability to think and work with numbers easily and to understand their uses (counting, measuring, ordering and labeling) and relationships.)

Numerals: conventional symbols that represent numbers (e.g., “1” is the numeral for “one”)

Rote count: recite the names of the numerals in order or sequence (e.g., singing a counting song)

Count with understanding: attach a number name to a series of objects; to understand that the number spoken when tagging or touching the last object also identifies the total number in the group

Ordinal numbers: numbers that indicate the position of an object in a sequence (i.e., first, second, third)

Operations on numbers: basic number combinations and strategies for computing such as addition and subtraction

Quantity: how many units are in a set (i.e., an amount or the result of counting)

Everyday fractions: numbers that represent parts of whole objects in the child’s environment (e.g., half a sandwich)

One-to-one correspondence: linking a single number name with one object, and only one, at a time

Estimate: making an educated guess as to the amount or size of something

Content component: Geometry and spatial sense

Geometry: the area of mathematics that involves shape, size, position, direction and movement and describes and classifies the physical world we live in

Location: where an object is in space

Orientation: the position or arrangement of an object

Position: the place where an object or person is in relation to others

Attribute: a characteristic or feature of an object such as color, size, shape, weight and number of sides

Spatial sense: children’s awareness of themselves in relation to the people and objects around them; it includes knowing boundaries, arrangements and positions

Shape: the form of an object

Three-dimensional: objects that have length, width and depth; solid figures such as cubes, spheres and cylinders

Two-dimensional: objects that have length and width but not depth; shapes such as squares, triangles and circles

Content component: Patterns and relationships

Patterns and relationships (algebra): the primary objective is for young children to be able to identify and analyze simple patterns, extend them and make predictions about them

Match: to find two objects that have at least one characteristic in common

Sort: to place or assign objects in two or more groups on a basis of at least one characteristic

Regroup: to place or assign objects in two or more groups using a different characteristic than was used the first time the objects were grouped

Order: arrange objects or numbers to show a progressive increase or decrease of a specific characteristic

Relative difference: the specific characteristic that differs among a group of objects (e.g., size)

Pattern: a sequence of colors, shapes, objects, sounds or movements that repeats again and again in a regular arrangement; patterns are a way for young students to recognize order and to organize their world

Extend: continue a pattern beyond what is shown

Content component: Measurement

Measurement: young children's intuitive notions of comparing volume, area, length and other attributes that they will eventually learn to measure; involves decisions about how much or how long

Compare: think about same and different; describe the relationship between two or more objects

Measurable features: a characteristic or attribute of an object that can be quantified (represented with a number) such as size, shape, weight and number of sides

Sequence: an arrangement of events or actions in a progressive order over time

Content component: Exploring data

Exploring data: informal experience with data by collecting, organizing, representing and comparing the information

Data: information gathered to answer a question

Classify: sort or form groups by similar characteristics

Organize: to arrange information in order to see relationships, often using graphs and charts

Sources of definitions

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Number and operations

Uses number to show quantity.

This standard refers to the child’s increasing understanding that numbers represent quantities. It is an important component of number sense — the ability to understand how numbers are used and how they are represented as well as relationships among numbers.

There are three indicators for this standard:

Indicators The child ...	Examples The child ...
Shows interest in counting and quantity.	<ul style="list-style-type: none"> uses fingers to indicate the number (e.g., holds up five fingers to show age). repeats counting rhymes and singing games with numbers. counts familiar objects (e.g., family members, friends, toys) although not always accurately. asks how many.
Develops an increasing ability to rote count in sequence.	<ul style="list-style-type: none"> counts one to 10 or beyond.
Counts objects with understanding.	<ul style="list-style-type: none"> counts five items (e.g., blocks, crayons, cars) accurately. hands one to five objects upon request (e.g., hands you three potatoes when you say, “Joe, hand me three potatoes.”)

Teacher strategies to promote use of number to show quantity include:

- encouraging children to experiment with counting in ways that are meaningful to them.
- engaging children in singing counting songs.
- reading big books that feature counting or numbers (e.g., *Ten Black Dots*).
- modeling counting of objects or people in meaningful contexts (e.g., to determine how many children are at a table).
- providing opportunities for counting objects.
- modeling that we use one counting word/number for each object.
- asking “how many” questions.
- posting a number chart at children’s eye level and modeling how to use it.
- playing counting games.

Number and operations

Uses language to represent number of objects.

This standard reflects the child’s growing knowledge and use of vocabulary associated with number and quantity. It provides a building block for mathematical understanding and communicating mathematical ideas. It is important to remember that children can make mathematical observations and connections throughout the day and in all areas of the curriculum.

There are four indicators for this standard:

Indicators The child ...	Examples The child ...
Uses language to compare number (e.g., more/less, greater/fewer, equal to).	<ul style="list-style-type: none"> looks at his own and another child’s blocks and determines who has more blocks. compares raisins with a friend’s and decides they have the same amount. asks, “How many more do you have?”
Combines and names how many.	<ul style="list-style-type: none"> puts the red, yellow and blue crayons together and counts how many crayons there are. recognizes that three cars and two trucks is a total of five vehicles.
Separates and names how many.	<ul style="list-style-type: none"> participates in finger plays, songs or stories such as <i>Five Little Monkeys</i> or <i>Five Little Ducks</i> that use backward counting. plays with a plastic ball and bowling pins and can tell how many fell down and how many are left standing.
Explores everyday fractions.	<ul style="list-style-type: none"> says (although not always accurately), “I have a whole orange,” or “I have half an apple.”

Teacher strategies that promote the use of mathematical language include:

- choosing books involving counting and math concepts during shared reading (see the list in the Resources section for suggested titles).
- engaging children in identifying environmental print with numbers and making environmental print “math” books.
- giving children materials to make collections with and conversing with them about how they sort or classify objects and how many there are.
- engaging children in finger plays, songs and stories that involve counting (forward and backward).
- providing opportunities for counting objects and asking about quantities.
- modeling vocabulary for making comparisons and indicating quantity.
- helping children make mathematical connections at snack time (e.g., asking, “Are there enough crackers for everyone? How many do we need?”).

Number and operations

Solves problems using number.

This standard refers to a child's ability to identify and solve personally meaningful problems involving number. As children begin to work with numbers to solve problems that are of interest to them, they deepen their understanding of what numbers represent as well as of the relationships among numbers.

There are four indicators for this standard:

Indicators The child ...	Examples The child ...
Names how many there are in a group (up to five objects) without counting.	<ul style="list-style-type: none"> recognizes that there are two or three crayons in a box. rolls a number cube and tells how many dots are on it without counting.
Uses one-to-one correspondence when counting objects.	<ul style="list-style-type: none"> can count five blocks in a row and say the number as each block is touched. gets a carton of milk for each child at the table. puts a cup with each napkin when setting the table. holds an additional finger up for each number when counting orally.
Uses one-to-one correspondence to compare the size of a group of objects.	<ul style="list-style-type: none"> compares two rows of blocks, two in one line and four in another, and can tell which one has more or less. matches number of cars to a friend's and says, "I have more."
Estimates, then counts to verify the number of objects.	<ul style="list-style-type: none"> while playing in the sand guesses how many cups it would take to fill a bucket and counts the cups of sand put in the bucket. guesses how many pennies are on the table, then counts the pennies.

Teacher strategies that promote problem solving with numbers include:

- taking advantage of natural contexts during the day (e.g., at snack time or when taking attendance) to encourage problem solving with numbers.
- showing children a glimpse of a small group of items (i.e., 1 to 3) and asking them how many there are.
- supplying the math center with a variety of math games, including teacher-made games, for children to explore.
- engaging children in all kinds of group games (e.g., aiming games, hiding games, guessing games, board games).

- encouraging children to make up board games or make their own rules for games.
- providing opportunities for children to estimate amounts and test their predictions.
- supporting children in making mathematical observations and connections in the block area and other centers.
- having children vote in appropriate contexts (e.g., in determining the name of the class pet).
- describing real-life situations involving numbers and a problem for children to solve (e.g., saying, "We have 10 cartons of milk for snack but only six straws. How many more straws do we need?").



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Number and operations

Uses numerical representation.

This standard refers to the child's ability to identify and distinguish numerals as well as develop an understanding of ordinal numbers (i.e., numbers that indicate where an object is positioned in a sequence). This skill is another building block of number sense and provides a means for children to communicate mathematical ideas.

There are five indicators for this standard:

Indicators The child ...	Examples The child ...
Uses drawings to represent number.	<ul style="list-style-type: none"> draws pictures showing size (e.g., short/tall) and quantity of family members. creates a way to keep score during a game. draws a picture to indicate number of objects or snacks.
Identifies numerals in everyday situations.	<ul style="list-style-type: none"> selects numerals on the telephone, calculator or computer. finds and names numerals in books or on signs.
Uses ordinal numbers (i.e., first, second, last).	<ul style="list-style-type: none"> can identify position in a line of children (e.g., who is first, second, last). can put three objects in a line and tell you which object is first, middle or last. tells the position of objects (i.e., first, second, last).
Writes some numerals.	<ul style="list-style-type: none"> draws numerals in sand. creates numerals with rolled clay or pipe cleaners. tries to write how old he or she is. tries to copy a telephone number.
Matches numeral with quantity.	<ul style="list-style-type: none"> when playing a game with a spinner or number cube, correctly counts the spaces on the game board that match the numeral or symbol. uses magnetic or flannel numerals to show how many marbles there are.

Teacher strategies for promoting use of numerical representation include:

- creating a classroom environment that contains many examples of how numbers are found and used in daily living (e.g., having phone books, receipts, etc., in the writing center; providing a calculator or adding machine in the math center).

- providing developmentally appropriate math software on the computer and having it accessible to children.
- encouraging children to represent numbers in many different media.
- having the children collect environmental print involving number.
- providing opportunities for children for tallying.
- providing resources and opportunities for children to experiment with writing numbers in the writing and art centers.
- providing time and space for children to play a variety of math games.
- posting number lines or charts at children's eye level for their reference.
- modeling the vocabulary of ordinal numbers.
- providing many opportunities for children to represent quantities with objects and exchange their ideas with their peers.

create a classroom environment that contains many examples of how numbers are found and used in daily living



Geometry and spatial sense

Investigates positions and locations.

This standard concerns the child’s developing understanding of the relationships between and among objects in the physical world as well as of the child’s own spatial sense: the consciousness of where he/she is in relationship to other people and objects.

There are three indicators of this standard:

Indicators The child ...	Examples The child ...
Takes objects apart and puts them together.	<ul style="list-style-type: none"> • builds with interlocking blocks. • puts lids on containers. • completes simple puzzles.
Uses actions and words to indicate position and location.	<ul style="list-style-type: none"> • moves self to show positions during play (e.g., under a table, in the tent, between friends). • uses objects to show position (e.g., puts the bears on/off/on top of/above/below/beside the box). • talks about objects that are on/off/under/in front of/behind/inside/outside/next to/ between/etc. • says when reading <i>The Three Billy Goats Gruff</i>, “The big billy goat is on the bridge, and the troll is under the bridge.”
Uses actions and words to indicate movement and orientation.	<ul style="list-style-type: none"> • moves self to show positions (e.g., up, down, forward, backward, around, through, to, from, sideways, across, back and forth, in a straight or curved path). • explains where objects in a room have been moved. • describes how to get to a location using landmarks. • follows a path or moves through an obstacle course. • draws paths or beginnings of a map to show location during play.

Teacher strategies to support children’s investigation of positions and locations include:

- having a variety of simple puzzles accessible to children.
- providing simple machines or objects for children to explore that can be taken apart and put together again.
- supplying the classroom with a variety of games and manipulatives that involve interlocking parts (e.g., *Don’t Break the Ice*, LEGOs, unifix cubes).
- providing an ample supply of blocks, in all shapes and sizes, as well as

read aloud texts
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adequate space and time for children to play with them and to discuss their actions and creations.

- modeling the vocabulary associated with position and location.
- reading aloud texts that involve changing positions and locations and engaging children in conversations related to these concepts.
- engaging children in movement and hiding games.
- supporting children’s attempts to make pictures or models of objects.
- encouraging children to make maps of their environment or to trace paths that they can then represent in pictures with landmarks.



Geometry and spatial sense

Explores shapes in the environment.

This standard refers to children’s growing awareness and identification of shapes in the environment as well as of dimensions. These abstract concepts become meaningful as children begin to explore shapes and observe their characteristics.

There are four indicators of this standard:

Indicators The child ...	Examples The child ...
Investigates and talks about the characteristics of shapes.	<ul style="list-style-type: none"> • says, “A circle is round.” • discovers that some blocks stack and some blocks roll. • says that squares and triangles have corners and straight sides.
Creates and duplicates three-dimensional and two-dimensional shapes using a variety of materials	<ul style="list-style-type: none"> • uses blocks to make other shapes or objects. • makes shapes with Play-Doh, pipe cleaners, string or yarn. • attempts to draw shapes and make pictures using shapes. • says, after cutting the sandwich, “Look, I made a triangle (or rectangle) with my sandwich.”
Identifies and names some shapes.	<ul style="list-style-type: none"> • points to or names simple shapes (e.g., box shape, ball shape, circle, triangle, square). • says, “The pizza is round. My piece is triangle-shaped.” • says, “The flag is the shape of a rectangle.”
Indicates if shapes are alike or different using one or more characteristics	<p>Three-dimensional shapes</p> <ul style="list-style-type: none"> • says, “A bubble and an orange are both like balls (spheres).” • says, “A block (cube) is shaped like a box.” • says, “This ball rolls, but this block does not.” <p>Two-dimensional shapes</p> <ul style="list-style-type: none"> • says, “A triangle has three sides,” or “A square has four sides.” • says, “A circle is curved (round) like a hula hoop.”

Teacher strategies to promote children’s exploration of shapes in the environment include:

- providing many opportunities for block play and supporting children’s conversations about their actions and experiments.
- asking open-ended questions about children’s constructions (e.g., saying, “Why do you think this block keeps falling off the tower?”).

- providing an art/construction area with a wide variety of materials for children to use to explore shapes.
- reading aloud and having available to children texts that illustrate or involve shapes; modeling the vocabulary associated with shapes.
- encouraging children to make observations and comparisons about shapes.
- providing opportunities for children to explore the attributes of shapes (e.g., in block play or the art area).

provide opportunities for block play
 — support children’s
 conversations about their
 actions and experiments



Patterns and relationships (algebra)

Recognizes relationships in the environment.

Children have a natural curiosity to explore their world and in their explorations begin to make all kinds of relationships, which is a reflection of mathematical thinking. As this thinking develops, children begin to sort, classify and recognize or make patterns, the foundation of algebra.

There are two indicators for this standard:

Indicators The child ...	Examples The child ...
Matches, sorts and regroups objects according to one or more characteristics.	<ul style="list-style-type: none"> • sorts plastic foods by size, color, shape or category. • matches objects that are alike (e.g., puts all of the two-hole buttons in one pile and four-hole buttons in another). • matches adult animals to their babies. • when playing Go Fish, matches all the cards with threes.
Orders things according to relative differences.	<ul style="list-style-type: none"> • sorts stuffed animals from smallest to largest. • talks about who is tall, taller, tallest. • arranges a group of blocks from longest to shortest.

Teacher strategies for promoting children’s recognition of relationships in the environment include:

- having an adequate supply of interesting materials accessible to children that they can sort and classify. These can be commercial products as well as recyclables and found objects, such as rocks, pine cones, etc.
- providing children many opportunities to sort and classify objects and to make small collections.
- engaging children in conversations about how they are sorting and classifying objects.
- playing games such as What’s My Rule? and doing activities that involve children in determining how groups are classified and in identifying patterns.

children can match
adult animals to their babies

Patterns and relationships (algebra)

Uses patterns in the environment.

This standard reflects the child’s growing ability to identify patterns. As the child begins to notice patterns in her surroundings, she also begins to explore making patterns in all kinds of ways, thus building algebraic thinking skills.

There are three indicators for this standard:

Indicators The child ...	Examples The child ...
Recognizes patterns.	<ul style="list-style-type: none"> • talks about color or pattern in clothing (e.g., says, “I have red and blue stripes on my shirt.”). • identifies color patterns that repeat (e.g., red, blue, red, blue).
Duplicates and extends patterns.	<ul style="list-style-type: none"> • imitates a pattern of sounds and physical movement (e.g., clap, stomp, clap, stomp ...). • continues rhythmic patterns. • completes the patterns in a story (e.g., says, “Brown Bear, Brown Bear, what do you see?”). • repeats a pattern according to size, color, shape, etc., while stringing beads. • predicts what comes next when an adult “reads” the pattern using simple vocabulary (e.g., car, car, boat, car, car, _____).
Creates patterns.	<ul style="list-style-type: none"> • creates simple patterns with beads or blocks according to color, size or shape. • creates simple patterns when drawing, coloring or painting.

Teacher strategies for promoting children’s use of patterns include:

- modeling and discussing patterns.
- engaging children in making body patterns.
- providing materials for children to use for pattern making (these can be recyclables and found objects as well as commercial products such as pattern blocks).
- reading aloud books that involve patterns and having children complete the patterns.
- engaging children in songs, dances, marches and clapping games that involve rhythmic patterns.
- using a pocket chart for creating and displaying patterns with numbers, letters, colors, shapes, etc., and engaging the children in predicting what comes next.

Measurement

Makes comparisons.

This standard refers to the child's increasing ability to make comparisons and identify sequences. This ability relates to both concrete objects and to concepts, such as time, that are very abstract to the child.

There are five indicators for this standard:

Indicators The child ...	Examples The child ...
Compares objects using measurable features.	<ul style="list-style-type: none"> • uses words to describe opposites (e.g., big/little, long/short, heavy/light). • chooses the largest snack. • says, "My bucket is heavier." • says, "This crayon is shorter."
Describes measurement.	<ul style="list-style-type: none"> • talks about an object being longer than another object. • uses a variety of language to describe measurement (e.g., shorter, taller, wider, bigger, heavier, lighter, holds more, hot, cold).
Orders three or more objects according to length or size differences.	<ul style="list-style-type: none"> • places ribbons in order by length. • puts cars in a row according to size. • puts pans (or measuring cups) inside each other.
Uses language associated with time in everyday situations.	<ul style="list-style-type: none"> • says, "Snack time comes after rest time." • says, "It's nighttime because it is dark." • says, "I eat breakfast in the morning." • says, "My birthday comes in the summer."
Anticipates, remembers and predicts a sequence of events.	<ul style="list-style-type: none"> • says, "I brush my teeth before I go to bed." • says, "We went to the library and then the grocery store." • recalls recent events and talks about them (e.g., says, "Yesterday we went to the zoo."). • describes the sequence of activities when going to the grocery store. • tells stories such as <i>The Three Little Pigs</i> with events in order. • points out when a familiar story is not told in the correct order.

Teacher strategies to encourage children in making comparisons include:

- modeling vocabulary for making comparisons (e.g., asking, "Is your train longer or shorter than Jennifer's?").
- engaging children in conversations where they are invited to make comparisons.
- reading aloud familiar stories and having children predict what comes next.
- engaging children in retelling familiar stories.
- providing opportunities for children to create "books" illustrating a sequence of events such as a field trip to the fire station.
- taking advantage of daily routines to introduce and model language associated with time.
- using a picture or pocket calendar for children to mark and track important events.

take
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time



Measurement

Uses measurement.

This standard refers to the child’s increasing ability to engage in the process of measurement. As children explore their environment and recognize that things have measurable properties such as length, height and weight, they learn that these properties can be assessed and compared. As they do so, they begin to implement both standard (e.g., rulers, scales) and nonstandard (e.g., yarn, hands) units of measurement.

There are two indicators for this standard:

Indicators The child ...	Examples The child ...
Explores ways to measure.	<ul style="list-style-type: none"> fills a container with solids or liquid (e.g., sand, ice cubes, water). pours liquid from one container to another container. sees how many blocks it takes to cover a sheet of paper.
Measures using objects.	<ul style="list-style-type: none"> places a string next to an object to measure length. uses the toy thermometer to measure the “patient’s” temperature. imitates using a ruler when helping dad.

Teacher strategies to promote children’s use of measurement include:

- providing different kinds of containers, measuring spoons, cups, balance scales, etc., at the sand or water tables for the children to explore (a tub of beans can also be used).
- asking questions that lead children to make comparisons that can be measured as they engage in center-time activities.
- using classroom situations to engage children in measuring (e.g., asking, “Do you think there’s enough room for the aquarium on that table? How can we find out?”).
- modeling how measuring tools (i.e., tape measures, rulers, etc.) are used and making them available to children to use for their own purposes.
- encouraging children to estimate amounts, lengths, etc., and then supporting their efforts to take measurements to compare with their estimates.

the child *imitates* using a ruler when helping dad

Exploring data

Collects, organizes and uses information.

This standard refers to children’s ability to collect and organize information for their own purposes. As children begin to sort, classify and compare as part of their daily lives, they start to find means to organize information, or data, in order to use it in meaningful ways. As children’s experience with collecting and organizing information increases, they also begin to explore ways to represent and interpret it (e.g., by participating in making charts or graphs).

There are three indicators for this standard:

Indicators The child ...	Examples The child ...
Asks questions to gather information.	<ul style="list-style-type: none"> asks, “What is your favorite color?” asks, “What month is your birthday?” asks, “What do you like to play outside?” asks, “How many brothers and sisters do you have?”
Sorts and classifies objects into groups and sometimes explains how the grouping was done.	<ul style="list-style-type: none"> puts objects together that have the same use (e.g., blocks, dishes, vehicles, clothes). explains how the buttons were sorted (e.g., says, “I put the red buttons together.”).
Evaluates information to answer questions.	<ul style="list-style-type: none"> says two kids have birthdays in July. says, “I have five trucks and four cars.” says, “More buttons are red.”

Teacher strategies to support children’s explorations with data include:

- encouraging children to ask questions to gather information.
- providing children many opportunities to sort and classify objects, to make small collections and to discuss how they are organizing them.
- engaging children in conversations about how they are sorting and classifying objects.
- using children’s interests to pose questions for them to investigate (e.g., saying, “At circle time, many children were talking about their pets. I wonder how many kinds of pets you all have? How can we find out?”).

Creating a Mathematically Rich Environment

When thinking about enhancing the environment to support children's mathematical thinking, it is important to remember that mathematics can extend across all curricular areas. Just as literacy can be introduced into many different activity centers, so can mathematics. Including math books in the reading center and rulers and graph paper in the writing center, for example, can encourage young children's mathematical thinking as can number charts posted at children's eye level. Supplying the classroom with plentiful and varied materials for children to explore also supports children's mathematical development and invites hands-on investigations and problem solving. A math center that includes a wide variety of math games, including teacher-made games, manipulatives, pattern blocks, etc., and a well-supplied block area can provide children with wonderful opportunities to extend their mathematical thinking. Providing a supply of found materials (e.g., rocks, shells, pine cones) for children to assemble small collections with is also helpful. In fact, as teachers organize their environment to take advantage of mathematical potential, it becomes clear that math is — or can be — everywhere!

supply the classroom with plentiful
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Involving Parents and Families

Partnering with parents and families to support children's mathematical development can be both rewarding and fun! Sharing math games to be played at home, or lending books in literacy bags that feature numbers and math concepts as well as story books are wonderful ways to involve families in the math curriculum. In addition, collecting environmental print or collecting information for class charts and graphs can turn into family projects. There are many resources for educators interested in involving parents in supporting their children's mathematical thinking, many of which are included in the next few pages. The Parents As Teachers program is also a wonderful resource; contact your school district for more information about this exemplary program.

Resources

Books:

Math

- Althouse, R. (1994). *Investigating mathematics with young children*. New York: Teachers College Press.
- Andrews, A., and Trafton, P. (2002). *Little kids — Powerful problem solvers: Math stories from a kindergarten classroom*. Portsmouth, NH: Heinemann.
- Burk, D., Snider, A., and Symonds, P. (1993). *Math excursions K: Project-based mathematics for kindergartners* (Rev. ed.). Portsmouth, NH: Heinemann.
- Burns, M. (1992). *Math and literature (K–3): Book one*. Sausalito, CA: Math Solutions.
- Charlesworth, R. (2000). *Experiences in math for young children* (4th ed.). Albany, NY: Delmar.
- Copley, J.V. (Ed.). (1999). *Mathematics in the early years*. Reston, VA: National Council of Teachers of Mathematics.
- Copley, J.V. (2000). *The young child and mathematics*. Washington, DC: National Association for the Education of Young Children.
- Dacey, L.S., and Eston, R. (1999). *Growing mathematical ideas in kindergarten*. Sausalito, CA: Math Solutions.
- Dacey, L.S., and Eston, R. (2002). *Show and tell: Representing and communicating mathematical ideas in K–2 classrooms*. Sausalito, CA: Math Solutions.
- Fosnot, C.T., and Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition, and subtraction*. Portsmouth, NH: Heinemann.
- Hill, D.M. (1977). *Mud, sand, and water*. Washington, DC: National Association for the Education of Young Children.
- Hirsch, E.S. (Ed.). (1995). *The block book* (3rd ed.). Washington, DC: National Association for the Education of Young Children.
- Kamii, C. (1982). *Number in preschool and kindergarten: Educational implications of Piaget's theory*. Washington, DC: National Association for the Education of Young Children.
- Kamii, C. (1985). *Young children reinvent arithmetic: Implications of Piaget's theory*. New York: Teachers College Press.
- Kamii, C., and DeVries, R. (1980/1996). *Group games in early education: Implications of Piaget's theory*. Washington, DC: National Association for the Education of Young Children.
- McGowen, D., and Schrooten, M. (1997). *Math play!* Charlotte, VT: Williamson.
- Moomaw, S., and Hieronymus, B. (1998). *More than counting: Whole-math activities for preschool and kindergarten*. St. Paul, MN: Redleaf Press.

- Moomaw, S., and Hieronymus, B. (1998). *Much more than counting: More math activities for preschool and kindergarten*. St. Paul, MN: Redleaf Press.
- Sheffield, S. (2002). *Math and literature (K–3): Book two*. Sausalito, CA: Math Solutions.
- Smith, S.S. (2001). *Early childhood mathematics* (2nd ed.). New York: Allyn and Bacon.
- Stenmark, J.K., and Coates, G.D. (1997). *Family math for young children*. Berkley, CA: University of California.
- Waite-Stupiansky, S., and Stupiansky, N.G. (1992). *Learning through play: Math, a practical guide for teaching young children*. New York: Scholastic.
- Wolf, D.P., and Neugebauer, B. (1999). *More than numbers: Mathematical thinking in the early years*.

General curriculum

- Bredenkamp, S., and Rosegrant, T. (Eds.). (1995). *Reaching potentials: Vol. 2. Transforming early childhood curriculum and assessment*. Washington, DC: National Association for the Education of Young Children.
- DeVries, R., Zan, B., Hildebrandt, C., Edmiaston, R., and Sales, C. (2002). *Developing constructivist early childhood curriculum: Practical principles and activities*. New York: Teachers College Press.
- Gestwicki, C. (1999). *Developmentally appropriate practice: Curriculum and development in early education* (2nd ed.). Albany, NY: Delmar.
- Green, M.D. (1998). *Themes with a difference: 228 new activities for young children*. Albany, NY: Delmar.
- Murphy, D.G., and Goffin, S.G. (Eds.). (1992). *Understanding the possibilities: A curriculum guide for Project Construct*. Jefferson City, MO: Missouri Department of Elementary and Secondary Education.
- National Association for the Education of Young Children. (1990). *Guidelines for appropriate curriculum content and assessment in programs serving children ages 3 through 8*. Washington, DC: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Assessment

- Helm, J.H., Beneke, S., and Steinheimer, K. (1998). *Windows on learning: Documenting young children's work*. New York: Teachers College Press.
- Leonard, A.M. (1997). *I spy something: A practical guide to classroom observations of young children*. Little Rock, AR: Southern Early Childhood Association.
- MacDonald, S. (1997). *The portfolio and its use: A road map for assessment*. Little Rock, AR: Southern Early Childhood Association.

Web sites:

- Association for Childhood Education International: www.udel.edu/bateman/acei
- Center for Innovations in Special Education: www.coe.missouri.edu/~mocise
- Child Development Associate (CDA) National Credentialing Program: www.nccc.org/Evaluation/cdacb.html
- Children's Defense Fund: www.childrensdefense.org
- Conference on Standards for Preschool and Kindergarten Mathematics Education: www.gse.buffalo.edu/org/conference/
- Connections Newsletters: www.canr.uconn.edu/ces/child
- Early Childhood Education on Line: www.ume.maine.edu/ECEOL-L
- Early Childhood Educators' and Family Web Corner: users.stargate.net/~cokids
- Early Childhood News: www.earlychildhoodnews.com
- ERIC/EECE Clearing House on Elementary and Early Childhood Education: ericece.org/publications.html
- Everything for Early Childhood Education Preschool-Grade 2: www.edupuppy.com
- Licensing of Child-Care Facilities in Missouri: www.health.state.mo.us/LicensingAndCertification/Dcc-gh.html
- Math and Children's Literature: www.carolhurst.com/subjects/math/math.html
- Math Forum: mathforum.org/library/levels/p/
- Missouri Department of Conservation: www.conservation.state.mo.us/teacher
- National Association for the Education of Young Children (NAEYC): www.naeyc.org
- National Association for Family Child Care: www.nafcc.org
- National Child Care Information Center: www.nccic.org
- National Council of Supervisors of Mathematics: ncsmonline.org
- National Council of Teachers of Mathematics: www.nctm.org/
- "On-Lion" for Kids: www2.nypl.org/home/branch/kids/
- Project Construct National Center: www.projectconstruct.org

- U.S. Department of Education, Office of Educational Research and Improvement and National Institute on Early Childhood Development and Education: www.ed.gov/pubs/EarlyMath/

Magazines:

- *Early Childhood News* (6 issues per year)
330 Progress Road
Dayton OH 45449
1-800-607-4410
- *Scholastic Early Childhood Today* (8 issues per year)
P.O. Box 54814
Boulder CO 80322-4814
1-800-544-2917
- *Young Children* (6 issues per year)
National Association for the Education of Young Children (NAEYC)
1509 16th Street, NW
Washington DC 20036-2460
1-800-424-2460

Other materials:

- NAEYC brochure for parents: *More than 1, 2, 3: The real basics of mathematics*. (McCracken, J.B.) No. 575. (See above address for ordering.)
- U.S. government booklet for parents: *Early Childhood: Where Learning Begins Mathematics*. (Print copies from www.ed.gov/pubs/EarlyMath/.)

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