

Lesson 3: Exponential Functions

Time: One- two class periods (50 minutes each)

Grade-Level Expectations Addressed:

A1C10 Compare and contrast various forms of representations of patterns.

A1D10 Understand and compare the properties of linear, quadratic, and exponential functions (include domain and range).

Essential Questions to Guide the Unit and Focus Teaching and Learning:

1. What is the relationship between geometric sequences and exponential functions?
2. How do you convert a recursive relationship into an explicit function?
3. How can technology help us analyze sequences and functions?

Specific Classroom Arrangement/Preparations:

Students assigned to groups in a classroom with computer access.

Lesson Materials:

- See technology/resources below

Technology/Manipulatives/Resources:

- Graphing Calculator (TI-83)
- Computers with Excel or other spreadsheet software

Step by Step Process:

LEARNING ACTIVITIES	QUESTIONS FOR STUDENTS	TEACHER SUPPORT
<p>Warm Up Activity: (See Below for Discussion/Review) Determine the missing terms of this sequence. 3, 6, 12, ____, 48, ____, ____, ____</p> <p>3, 6, 12, <u>24</u>, 48, <u>96</u>, <u>192</u>, <u>384</u>, <u>768</u> What is the name of this type of sequence? A geometric sequence What would be the 30th term? 1,610,612,736 What methods can be used to find this term or any other term? $g_{30} = 3 \cdot 2^{29} = 1,610,612,736$ or complete the sequence through 30 terms: (see the Write an explicit and recursive formula to generate the terms of the sequence. Explicit formula: $g_n = 3(2)^{n-1}$ Recursive formula: $g_{n+1} = (g_n)2$ and $g_1 = 3$</p>	<p>What is the pattern that seems to be established by this sequence of numbers? How do you find the nth term?</p>	<p>Display the warm up activity so that students can begin working on it as they come into the room. Give students 5-10 minutes to do the warm up activity</p>
<p>Discussion/Review for Warm Up Activity:</p> <p>Definitions: An arithmetic sequence is a sequence in which the difference of any two consecutive terms is constant. This number is called the common difference.</p> <p>A recursive formula for an arithmetic sequence would have the initial term, a_1, and a formula for the nth term involving the previous term and the common difference d. The formula would be $a_n = a_{n-1} + d$.</p>		<p>Review these definitions as you discuss the warm up activity. Some textbooks use Now-Next notation which is addressed later in this lesson, but it could be</p>

An **explicit formula** for an **arithmetic sequence** would have a formula for the n^{th} term involving the initial term, the common difference d and the term number n . The formula would be $a_n = a_1 + (n - 1)d$. This explicit function is also called a **linear function**.

A **geometric sequence** is a sequence in which the ratio between consecutive terms is a constant. This constant is called the common ratio, r . (i.e. $a_{n+1}/a_n=r$)

A **recursive formula** for a **geometric sequence** would have the initial term and a formula for the n^{th} term involving the previous term and the common ratio r . The formula would be $a_n = (a_{n-1})r$ and a_1 .

An **explicit formula** for a **geometric sequence** would have a formula for the n^{th} term involving the initial term, the common ratio r and the term number n . The formula would be $a_n = a_1 \cdot r^{n-1}$. This explicit formula is also called an **exponential function**.

Step 2: Looking at a geometric sequence from a graphical approach

x	y
1	3
2	6
3	12
4	24
5	48
6	96

What is an arithmetic sequence?
 What is a geometric sequence?
 How do they differ?

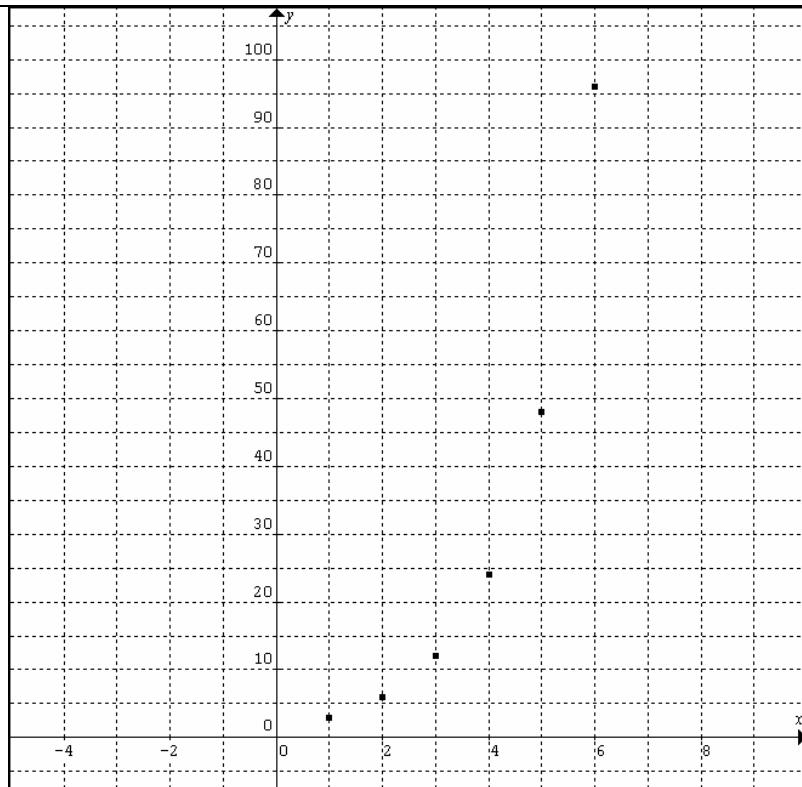
discussed here, also.

Let's look at a graph of the given points:

 Before we graph them, what do you expect the graph will look like?

 Why?

Students can recognize exponential functions because there is a constant ratio between y-values if there is a constant change in x. Examples of this ratio might be the y-value is multiplied by 3 every time x increases by 4, or the y-value is multiplied by $\frac{1}{2}$ every time x increases by one, etc.



This is the graph of the data points from the given table.

Were our predictions correct?

What could we have noticed to make our predictions correct?

The y-value doubles each time x increases by 1.

Step 3: Small Group Work

Can you create a situation that would produce this pattern?

What does x and y represent?

Spend 5 minutes in small groups answering these two questions.

Let each group present its scenario for the graph.

One possible scenario: (Let x represent the day, and y represent the number of volunteers.) Three friends, Larry, Moe and Curly decide to volunteer at the homeless shelter one day. The day is so rewarding the three friends come back to volunteer the next day, and each brings a friend to volunteer as well. The next day those six return, and each brings a friend. This pattern continues. How many volunteers will there be on the sixth day.

Based on the scenario the students like best, produce an equation that will model the scenario.

Step 4: Develop explicit exponential functions from a recursive approach:

Begin with a recursive definition:

Recursive:

$$a_1 = \text{first term}$$

$$a_{n+1} = (a_n)2$$

This is also sometimes stated with Start and

Next-Now notation:

$$\text{Start} = 3$$

$$\text{Next} = \text{Now} \cdot 2$$

Then find the explicit function:

$$y = 3 \cdot 2^{x-1} \quad \text{or} \quad \text{number of volunteers} = 3 \cdot 2^{\text{numberofdays}-1}$$

How many volunteers would there be on day 0 according to this pattern?

Reversing the pattern, the number of volunteers on day 0 would be half of the number on day 1 or 1.5 volunteers. Thus, $y = 1.5 \cdot 2^x$ is equivalent to the above.

Let's verify this algebraically:

$$y = 3 \cdot 2^{x-1} = 3 \cdot 2^x \cdot 2^{-1} = \frac{3}{2} \cdot 2^x = 1.5 \cdot 2^x$$

For our scenario 1.5 volunteers is NOT sensible.

What is one possible scenario for the data points given in the table?

Try to get students to approach the scenario in this way:

The number of volunteers = $3 \cdot 2 \cdot 2 \cdot 2 \cdot \dots$. You start with 3 volunteers on day 1 and each succeeding day the number of volunteers is two times the number of volunteers on the previous day. This will help with the relationship between explicit and recursive functions.

According to the pattern established by our formulas, what is y if x = 0? Does this make sense for the scenario you have chosen?

Paper Folding Activity

1. How thick is a piece of paper?

Measure (or estimate) the thickness of a ream of paper and find the thickness of one sheet from this. **Approx. 2 in/500 sheets = .004 in/sheet**

2. How thick is it when it has been folded in half once? .008 in.

3. Twice? .016 in.

4. Three Times? .032 in.

5. Ten Times?

$$.004 \cdot 2^{10} = 4.096 \text{ in}$$

6. Fifty Times?

$$.004 \cdot 2^{50} = 4.5036 \times 10^{12} \text{ in}$$

$$= 375299968948 \text{ ft} (\div 12)$$

$$= 71,079,539.5734 \text{ miles} (\div 5280)$$

Give all answers in a reasonable unit of measurement. Convert to a different unit as necessary. Provide the work that shows how you obtained your answers.

US Census Data Problem

This data comes from

<http://www.census.gov/statab/hist/02H S0001.xls>

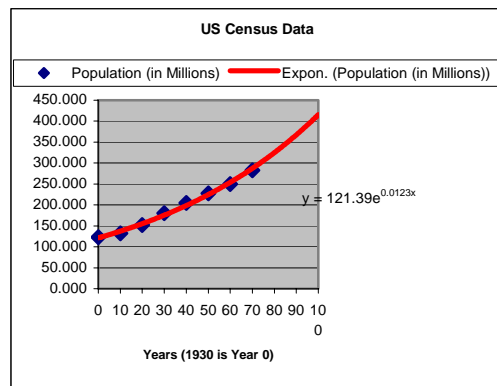
This is census data for the US population since 1930 (year 0). Plot this data and use an exponential regression to estimate the population for the year 2030.

See Appendix for the [Paper Folding Activity](#) and the [National Debt Activity](#), as well as the [scoring guide](#) for the paper folding activity.

Answers:
The population in 2030 is projected to reach about 420

Year	Population (in Millions)
0	123.188
10	132.122
20	152.271
30	180.671
40	205.052
50	227.726
60	250.132
70	282.434

How accurate do you feel your prediction is? What might affect the accuracy of your prediction?



million people. The prediction would appear to be very accurate based on the population trend established since 1930. If various measures were instigated to control the population then this prediction might be too high. Also, various national disasters that cause large losses of life could affect the population.

Additional physical world data sets that model the exponential function:

National debt data, AIDS data, Money Growth and Investments, Ball bouncing,

		Creation of/and Analyzing Exponential Scatterplots in Excel or on a graphing calculator, Richter Scale, earthquakes, Bacteria Growth, Medicine in Blood Stream, and Population Growth
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