Measurement

Kindergarten

BIG IDEA (1): Understand measurable attributes of objects and the units, systems and processes of measurement

	CONCEPT	EXPECTATION	EXAMPLE
A	Determine unit of measurement	Compare and order objects according to their size and weight.	Problem: Give each student a paper clip then have them get a crayon out of their box of crayons. As students compare the two sizes, ask the following questions. a. Hold up the item that is longer. b. How did you figure out which one was longer? Answer: a. More than likely the crayon will be the longer. b. Students should indicate that they have aligned the endpoints of the crayon and the paper clip to compare them. Problem: Students will explore then organize weights of different objects. This would include experiences using a pan balance. Have students demonstrate the compared weight of two objects by using their hands to represent a pan balance. The teacher selects two items with noticeably different weights, such as a book and a piece of paper. Show them in which hand you will be holding each of the items. Have them predict which hand will go down to show which item is heavier. As you hold the two items in your hands, exaggerate the drop of your hand to help the students indentify which item is heavier. As additional questions similar to the following: a. What happens if I put a paper in one hand and a book in the other? b. What would happen if I switched the paper and book from one hand to

CONCEPT	EXPECTATION	EXAMPLE
		the other? c. If I hold my shoe in one hand and a scarf in the other hand, what do you think will happen? Answer: a. The heavier item will go down (or other valid response). b. The other (or opposite) hand would go down. c. Your hand with shoe would go down because it is heavier than the scarf. Students will organize classroom objects (students, pencils, crayons, etc.) according to their size or weight. <u>TEACHER NOTES</u>: "Measurable attributes are quantifiable characteristics of objects. Recognizing which attributes of physical objects are measurable is the starting point for studying measurement, and very young children begin their exploration with measurable attributes by looking at, touching, and comparing physical things directly."[1] Teachers should begin with comparing two objects, such as the length of two crayons, and increase the number of items depending on students' readiness. In addition to developing and understanding of measurement concepts, they should at the same time develop the vocabulary to describe them, such as longer, shorter, heavier, lighter, most, least, etc.

[1] *Navigating through measurement in grades Prekindergarten-Grade 2* (p. 12.) (2003). Reston, VA: National Council of Teachers of Mathematics

CONCEPT	EXPECTATION	EXAMPLE
B Tell and use units of time	Describe passage of time using terms such as today, yesterday, tomorrow	Problem: Distribute copies of the current month calendar to students. a. Point to day on the calendar and ask students to color tomorrow on the calendar in red. b. Ask students to color yesterday in blue on the calendar. c. As students to color today's date in green. Answers: Answers will vary according to the day of the week the teacher chooses to start with. <u>TEACHER NOTES</u>: Students will be learning the days of the week and their correct order. Begin each week by talking about activities that will be occurring on different days of the week, marking them with stickers, etc. then refer to the calendar throughout the week to talk about today, yesterday, and tomorrow.

CONCEPT	EXPECTATION	EXAMPLE
C Count and compute money	Identify and know the value of a penny, nickel and dime.	Problem: The teacher reads each statement and students hold up the correct coin. a. I am the same value as 5 pennies. b. I have a value of 1 cent. c. I have the same value as 10 pennies. Answers: a. nickel b. penny c. dime Problem: Distribute to each student 12 pennies. The teacher shows the students a coin (penny, nickel, or dime) and asks students to count out the number of pennies on their desk that the coin represents.

BIG IDEA (2): Apply appropriate techniques, tools and formulas to determine measurements

CONCEPT	EXPECTATION	EXAMPLE
A Use standard of non-standard measurement	Measure with multiple copies of a unit of the same size (e.g., paper clips laid end to end)	Problem: Have each student stand on a piece of paper while the teacher traces around their shoe. (If tracing around children's should will take too long, the teacher may want to consider using pictures of various size die cut shoes.) Then have each student take the picture of their shoe and find the length of their shoe using each of the units of measure placed end to end. a. small paper clips b. large paper clips c. pennies d. dimes e. quarters Answer: Answers will vary depending on the size of shoes and units used to measure them. <u>TEACHER NOTES:</u> "Linear measure is the most visible attribute of an object. Initially, young students determine length by using multiple copies of a unit. The units are placed end to end until they line up to approximate the length of the object."[2]

[2] *Navigating through measurement in grades Prekindergarten-Grade 2* (p. 29.) (2003). Reston, VA: National Council of Teachers of Mathematics

MEASUREMENT

Grade 1

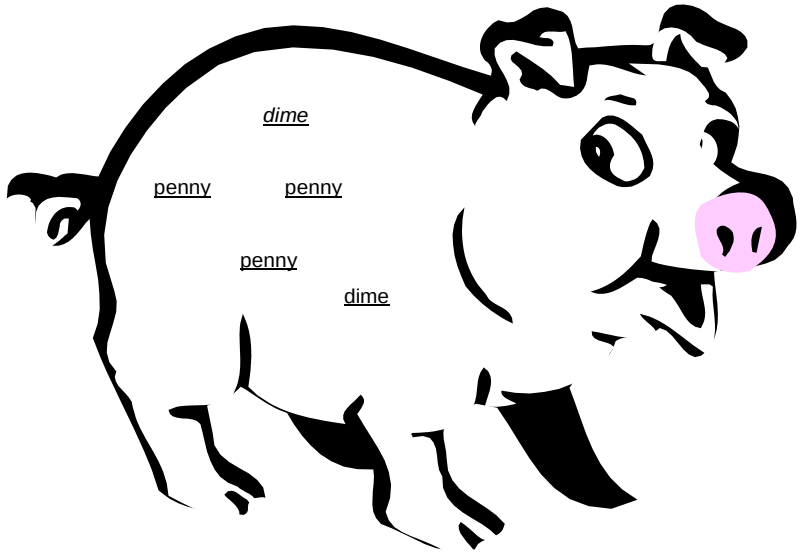
BIG IDEA (1): Understand measurable attributes of objects and the units, systems and processes of measurement

	CONCEPT	EXPECTATION	EXAMPLE
A	Determine unit of measurement	Select the appropriate tool for the attribute being measured.	Problem: Ask students to choose the appropriate tool from a paper clip, a bean, a meter stick, and a 12 inch ruler to measure the distance from the floor to the door knob. Then ask, why did you choose that tool? Answer: While any of the tools could be used, the meter stick would be the easiest, most efficient way to measure the distance from the floor to the door knob. TEACHER NOTES: The teacher will select items in the classroom or school and have students determine which tool or measurement would be appropriate. Include standard and non-standard tools of measurement. Note: Students are not required to find the measure, only to find the appropriate tool for measuring.

DEFINITIONS:
attribute-a characteristic or distinctive feature—such as shapes, size, color—of an object or given set of objects.[1]

[1] Eather, J. *A math dictionary for kids.* Retrieved June 5, 2004 from http://www.amathsdictionaryforkids.com

	CONCEPT	EXPECTATION	EXAMPLE
C	Tell time and use units of time	Tell time to the nearest hour	Problem: Which clock below shows 5:00 o'clock?  a. b. c. d. Answer: b. <u>TEACHER NOTES</u>: Using a set of classroom clocks provides opportunities for students investigate time. The teacher might show a clock with a certain time on it to the class and ask them to write down the time to the nearest hour. Or the teacher might distribute classroom clocks to students and ask them to show different times on their clock and discuss to determine the time to the nearest hour. Teachers can easily do this by periodically have the students do a time check on the hour. The teacher sets a timer or alarm to go off on the hour. When it the alarm sounds, have a student tell the time. Do this periodically throughout the day. Utilize classroom schedules to relate analog time to the written representation.

	CONCEPT	EXPECTATION	EXAMPLE
D	Count and compute money	Count money to fifty cents, including quarters and half dollars	Problem: Ann has the following coins in her pocket—2 pennies,1 dime, and1 nickel. How much money does she have? (You may use your coins to help you find the answer.) Answer: 17 cents Problem: The piggy bank shows the money that Tom has. How much money does he have? (You may use your coins to help you find your anwer.  Answer: 23 cents

		Problem: Mrs. Jones gave 48 cents for some candy. She used one quarter and some other coins to pay for the candy. What other coins did she use? Answers: 2 dimes and 3 pennies; 1 dime, 2 nickels, and 3 pennies; 4 nickels and 3 pennies; 3 nickels and 8 pennies; 2 nickels and 13 pennies; 1 nickel and 18 pennies; or 23 pennies. Problem: Ken wants 50¢ to pay for his treat. Show four different ways Ken could make 50¢ to pay for his treats. You may use your coins to help you find your answers. Answer: There are over 45 different ways to represent 50¢. Accept all correct combinations. <u>TEACHER NOTES:</u> Teachers should have students identify different coins and combinations of coins to equal fifty cents.

BIG IDEA (2): Apply techniques, tools, and formulas to determine measurements

	CONCEPT	EXPECTATION	EXAMPLE
A	Use standard and non-standard measurement	Use repetition of a single unit to measure something larger than the unit, (e.g., measuring the	Problem: How many does it take? The teacher should prepare ahead of time several outlines of objects on large pieces of paper including an adult shirt, t-shirt, a baseball bat, etc. Have students use standard units (such as12-inch rulers, meter sticks, etc.) and non-standard units (such as unsharpened pencils, chalkboard erasers, paper clips, etc.) to determine the length of each of the

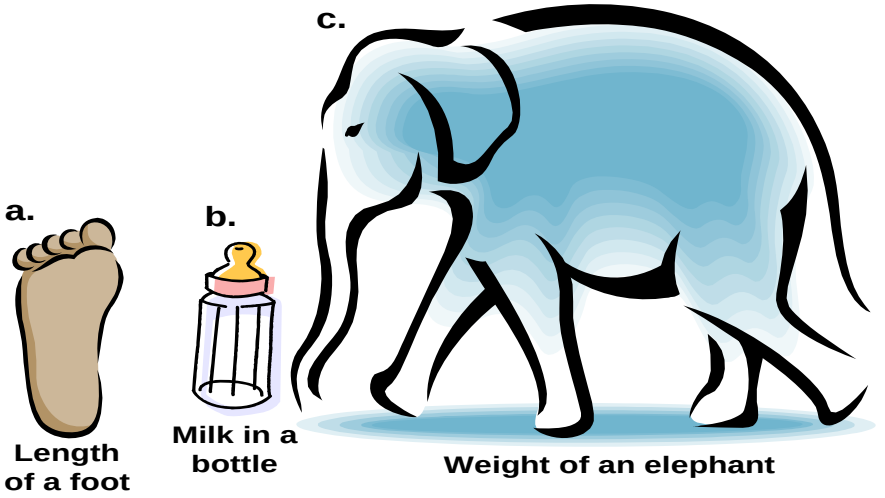
	length of the room with a single meter stick)	items drawn on paper. Answer: Answers will vary depending on the objects drawn on paper and repetitive units used for measuring. <u>TEACHER NOTES:</u> The teacher should model and demonstrate to students proper techniques for laying the measuring unit end to end to determine the length of the item being measured.

DEFINITIONS:

MEASUREMENT

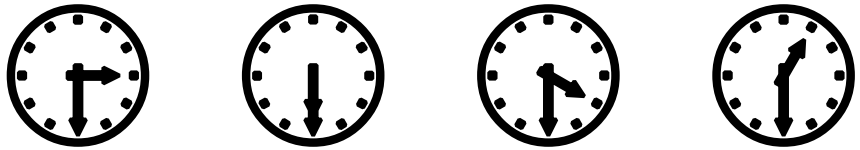
Grade 2

BIG IDEA (1): Understand measurable attributes of objects and the units, systems and processes of measurement

	CONCEPT	EXPECTATION	EXAMPLE
A	Determine unit of measurement	Select the appropriate tool for the attribute being measured.	Problem: Which tool, pound scale, inch ruler, or cup would be used to measure each of the following?  Answer: a. Inch ruler b. cup c. pound scale

		TEACHER NOTES: The teacher will select items in the classroom or school and have students determine which tool or measurement would be appropriate. Include standard and non-standard tools of measurement. Also include appropriate tools for measuring volume.

DEFINITIONS:
attribute-a characteristic or distinctive feature—such as shapes, size, color—of an object or given set of objects.[1]

	CONCEPT	EXPECTATION	EXAMPLE
C	Tell time and use units of time	Tell time to the nearest half hour	Problem: Use the clock pictures. A. B. C. D.  a. Which clock shows a time of 1:30? b. Which clocks shows a time of 6:30? c. Which clock shows a time that is half an hour after 4:00? Answer: a. D

[1] Eather, J. *A math dictionary for kids.* Retrieved June 5, 2004 from http://www.amathsdictionaryforkids.com

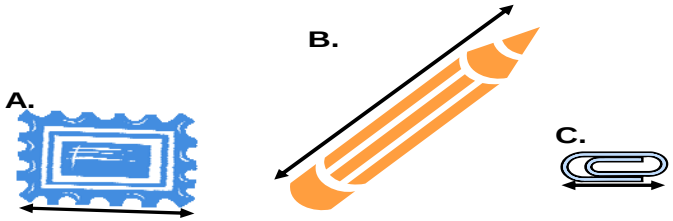
		b. B c. C TEACHER NOTES: Using a set of classroom analog clocks provides opportunities for students investigate time. The teacher might show a clock with a certain time on it to the class ask them to write down the time to the nearest half hour. Or the teacher might distribute the classroom clocks to students and ask them to show different times on their clock and discuss to the nearest half hour what the time is.

	CONCEPT	EXPECTATION	EXAMPLE
D	Count and compute money	Count money to a dollar	Problem: Tim has five coins that equal 78 cents. What coins does he have? Answer: One half dollar, 1 quarter, and 3 pennies. Problem: Lou Ann has eight coins that equal more than 85 cents. What coins does she have? Answer:

		Numerous answers are possible including: 5 dimes, 1 quarter, and 2 nickels; 6 dimes, 1quarter, 1 penny; 1 half dollar, 1 quarter, 2 nickels, and 4 pennies.

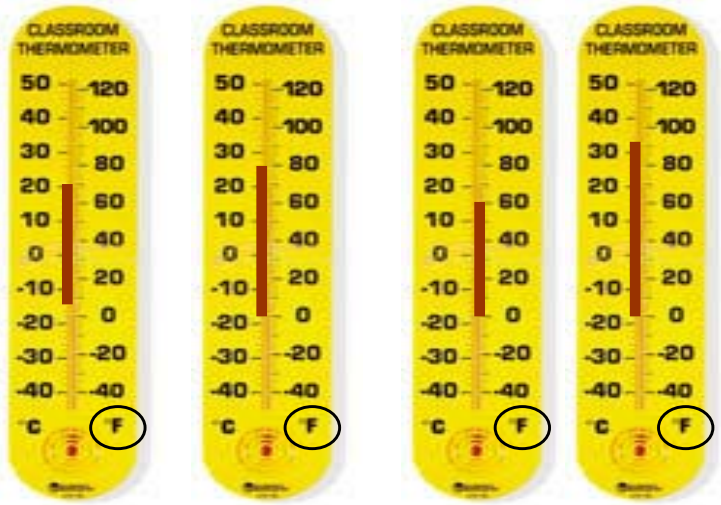
BIG IDEA (2): Apply techniques, tools, and formulas to determine measurements

	CONCEPT	EXPECTATION	EXAMPLE
A	Use standard and non-standard measurement	Use tools to measure (size, temperature, time, weight) to the nearest inch, centimeter, degree, hour and pound.	Allow students numerous opportunities to measure objects in the room using their centimeter ruler and their inch ruler. Problem: Direct half of the class to measure items with their centimeter ruler. Direct the other half of the class to measure the same items with their inch ruler. Answer: Discuss and compare the answers with the two different measurements. Problem: Use your centimeter ruler to measure each picture to the nearest centimeter.



Now, use your inch ruler to measure each picture to the nearest inch.
Answers:
Lengths of objects will vary depending on the size the teacher reproduces for students to measure.

Problem:



Day 1 2 3 4

NOTE Sukanya and Art: The thermometers I used are probably copyright so we need to replace them with thermometer graphics that are not copyrighted.

The second grade class recorded the Fahrenheit high temperature for four straight days.

a. What was the lowest temperature recorded?
b. What was the highest temperature recorded?

Answer:

a. 60 degrees Fahrenheit
b. 90 degrees Fahrenheit

TEACHER NOTES:

Teachers should provide ample opportunities for students to use tools to measure weights, lengths, and volumes of various objects.

		"By the end of grade 2, students should recognize that length, area, capacity, weight, and time can be measured. The should recognize that both a number and unit are needed to report a measure and that the size of the unit is related to the size of the number."[2]

DEFINITIONS:

[2] *Navigating through measurement in grades Prekindergarten-Grade 2* (p. 31.) (2003). Reston, VA: National Council of Teachers of Mathematics

MEASUREMENT

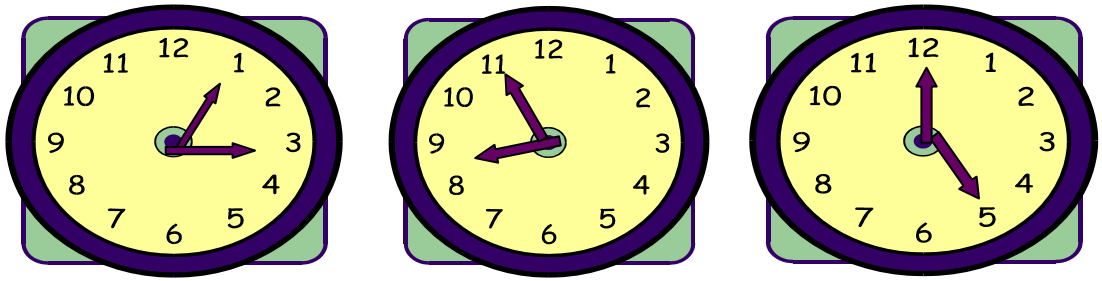
Grade 3

BIG IDEA (1): Understand measurable attributes of objects and the units, systems and processes of measurement

	CONCEPT	EXPECTATION	EXAMPLE
A	Determine unit of measurement	Identify, justify and use the appropriate unit of measure (linear, time, weight)	Problem: The teacher asked each student to measure the length of their math book. Which of the following units should they use to measure their book? a. inches b. feet c. yards d. miles Answer: a. inches Problem: The Adams family drove 30 _____ to visit the zoo. Which unit of measure should be used in the blank to show how far they traveled? a. inches b. feet c. yards d. miles Answer: d. miles Problem: Mrs. Bee bought a pumpkin at the store. The pumpkin weighed

		12_____. Which unit of measure should be used in the blank to show how much the pumpkin weighed? a. ounces b. pounds c. gallons d. tons Answer: b. pounds Note: As an extension to each problem above, the teacher might as students to explain why that particular unit makes sense. *TEACHER NOTES:* The understanding of unit is a complex process. First, students need to recognize that the unit of linear, time or weight measure must be related to the characteristic being measured. To measure the length of a desk, for example, they need to use linear units of measure, such as centimeters, inches for feet. To measure how long it takes to walk home from school they would need minutes or hours, which are units of time. To measure the weight of their pet, they would need to use ounces or pounds. After choosing the appropriate type of measurement, they also have to consider the most efficient unit within the type of measurement for measuring a particular length, time, or weight.

DEFINITIONS:

CONCEPT	EXPECTATION	EXAMPLE
C Tell time and use units of time	Tell time to the nearest five minutes	Problem: Use the clock pictures to help you answer the questions. A. B. C.  The third grade teacher took pictures of the clock throughout the school day. a. Which clock shows the earliest time? What time is it? b. Which clock shows the time closest to lunch? What time it is? Answer: a. B; 8:55 b. C; 12:25 Note: Please make an allowance for a difference of plus or minus a minute or two in the answers. <u>TEACHER NOTES</u>: Teachers can use daily schedules to utilize opportunities for students to tell time to the nearest five minutes. Throughout the day, the teacher can ask students to record times on assignments, etc. Using Judy clocks, have students identify various times to the nearest five minutes. One way to proceed is to • Have students identify the hour. • Ask students whether the minute hand is showing the number of minutes before or after the hour.

		• Have students count the minutes by five. • Ask students to state the time. Throughout this work, relate the students' recorded time to what could be happening at school or home at that time.

DEFINITIONS:

	CONCEPT	EXPECTATION	EXAMPLE
D	Count and compute money	Determine change from $5.00 and add and subtract money values to $5.00	Problem: Kelly gave the clerk a five dollar bill to pay for a notebook that totaled $3.15. How much change did she get from the $5? a. $1.85 b. $2.15 c. $2.85 d. $6.85 Answer: a. $1.85 Problem: Lance has $5. He wants to buy items (including tax) that cost $1.25, $0.56, and $2.75. a. Explain whether he has enough money to buy the items. b. What coins could he receive as change?

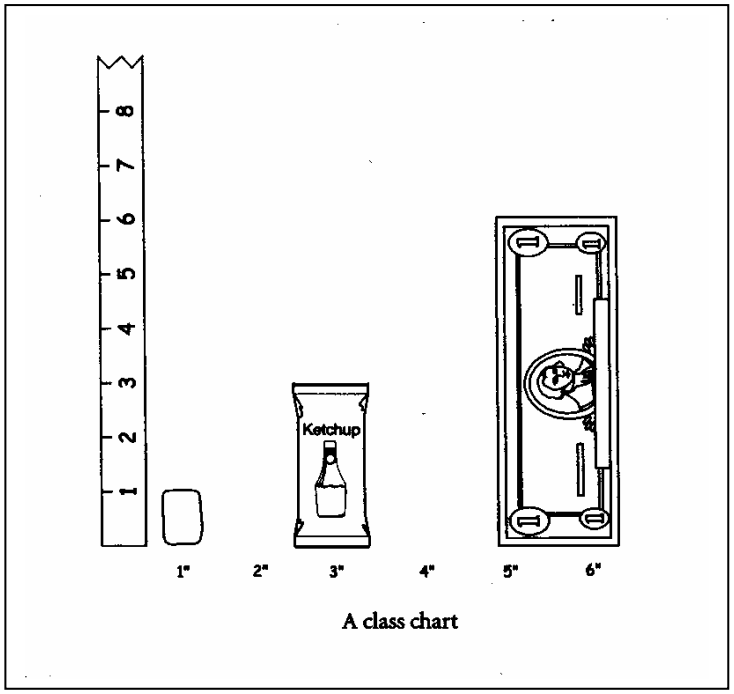
		Answer: a. $1.25 + $0.56 + $2.75 = $4.56. The total cost of the items is less than $5 so Lance does have enough money to buy the items. b. Accept any correct combination of coins with a value of 44¢. TEACHER NOTES: Have students take turns being storekeepers for classmates as they "buy" daily snacks with paper bills. Purchasers and storekeepers should both figure out the change before the storekeeper gives change to buyer. Create other situations in which students "buy" materials such as a limited number of art supplies for a project, etc.

BIG IDEA (2): Apply techniques, tools, and formulas to determine measurements

	CONCEPT	EXPECTATION	EXAMPLE
A	Use standard and non-standard measurement	Use a referent for measures to make comparisons and estimates	Problem: If  About how many  will cover the space below? Answer: About 12. Problem: Use the classroom wall chart (see below) to help you find the answer.

Would it take more sugar packs or small ketchup packs to measure the distance around the room? How do you know?

Answer:
It would take more sugar packs because they are shorter than the ketchup packs.



A class chart

TEACHER NOTES:
Students may develop their own personal referents which may consist of items from home as well as school (Joram, 2003). Joram also mentions the creation "measurement similes" (such as "a spider whose abdomen [is] 6 cm in diameter, about as wide as a lemon") to assist in developing a sense of measurement. However, she cautions that in developing personal referents, students may lose sight of the larger context, of how the referents are related to one another. Therefore, she suggests making a classroom chart with a yardstick attached to the left side and the number of inches written horizontally across the bottom. Above some of the inches listed on the bottom, students can attach their personal referents—a piece of bubble gum for one inch, a packet of ketchup for three inches, and a dollar bill for six

		inches. In this way the students are relating the benchmarks to one another and to the yardstick on the left.

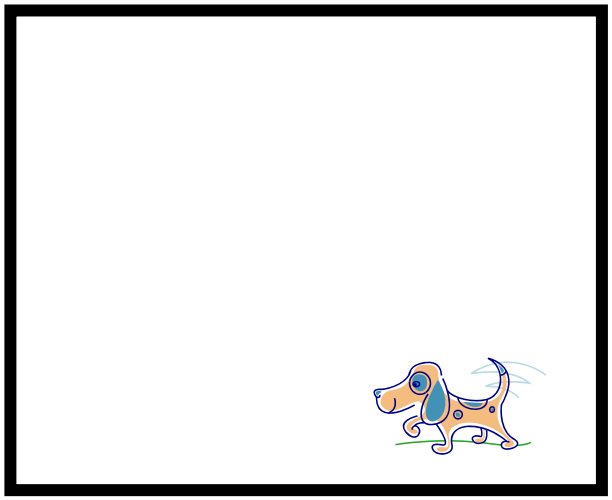
DEFINITIONS:
Referent—a familiar object or place that a student can use as a basis for estimating the measurement of something; students might think of the length of their desks, the size of an orange, etc. [1]

	CONCEPT	EXPECTATION	EXAMPLE
C	Apply geometric measurements	Determine the perimeter of polygons	Problem: Below is a picture of Ben's dog pen. What is the total distance around the pen? Show how you got your answer.

[1] Joram, E. (2003). *Benchmarks as tools for measuring sense*. Clements, D. H. & Bright, G. (EDds). *Learning and teaching measurement*. Reston, VA: National Council of Teachers of Mathematics, 2003. (p. 57 – 67)

5 feet

4 feet



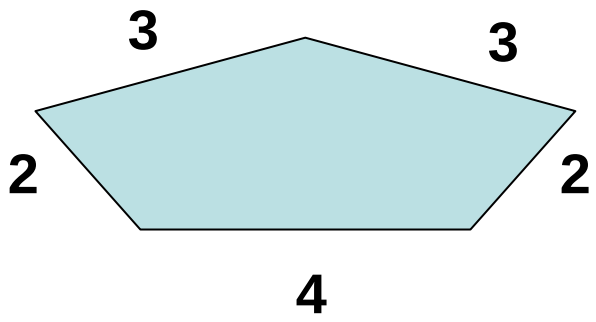
Answer:
I added 4+ 4 + 5+ 5= 18 feet; I add 4+ 5= 9 to get the two sides then added 9 + 9= 18 because there are four sides.

Problem:
The sides of a triangle measure 3 cm, 4 cm, and 5 cm. What is the perimeter of the triangle?

a. 6 cm
b. 12 cm
c. 24 cm
d. 60 cm

Answer:
b. 12 cm

Problem:
What is the perimeter of the figure below?



a. 9 units
b. 12 units
c. 13 units
d. 14 units

Answer:
d. 14 units

TEACHER NOTES:
It is important for students to have frequent experiences with actually measuring items. Besides helping children gain measurement skills, the following activities also help students develop number sense

- With a tape measure actually measure the distance around large polygon models. Record the total. Then measure each side, record the results, and find the total. While students' measurements may not be exactly the same in each case, children may be able to see patterns and create informal rules for finding perimeters.
- Imagine putting a border on the edge of a bulletin board. How long will the paper be for each side? What is the total length around the

		sides? Imagine that your classroom is a play yard for your pet dog. You want to put a fence around it. What is the length of the fence you will need to buy?

MEASUREMENT

Grade 4

BIG IDEA (1): Understand measurable attributes of objects and the units, systems and processes of measurement

	CONCEPT	EXPECTATION	EXAMPLE
A	Determine unit of measurement	Identify and justify the unit of linear measure including perimeter (customary and metric)	Fourth grade measurement should include both customary and metric units of measure. Students are **not** required to convert between the two systems. Instead we should build on their capacity to understand and use metric units of measure. Beginning activities should first be done with the familiar customary units of measure, then students should be given opportunities to use the appropriate metric unit for the same measure. Problem: What customary and metric unit would you use to measure each of the following? a. the length of a pencil b. the length of the football field c. the distance from your home to the White House Answers: a. inches; centimeters b. yards; meters c. miles; kilometers Problem: Luke and Seth are discussing what unit they should use to measure the length of their swimming pool. Luke says they should use inches and Seth says they should use feet. Explain which unit of measure you think they should use.

		Answer: I think they should use feet because it is a bigger unit and they can measure it more quickly if they measure it in feet. *<u>TEACHER NOTES:</u>* Carefully observe students' measuring strategies. Some students may need to use several rulers or meter sticks and line them against their objects. In doing so, they should learn how to place the first ruler against the beginning of the object and to arrange subsequent rulers so there are no gaps between them. Gradually students will use a single ruler, systematically repositioning it until the whole length is measured. In all cases students must recognize the importance of beginning the process with the zero point on the ruler, even if zero isn't marked. (Or, if they begin at a different point on the ruler, in their calculations they'll need to compensate for that positioning.)

DEFINITIONS:

	CONCEPT	EXPECTATION	EXAMPLE
B	Identify equivalent measures	Identify equivalent linear measures within a system of measurement	Problem: A meter is the same as: a.1 centimeter b. 10 centimeters c. 100 centimeters d. 1000 centimeters Answer: d. 100 centimeters

		Problem: The tallest student in the class is 60 inches tall, this is the same as: a. 3 feet b. 4 feet c. 5 feet d. 6 feet Answer: c. 5 feet <u>TEACHER NOTES:</u> When reporting measurements, students need to remember that it is essential to give the unit as well as the numerical value.

DEFINITIONS:

	CONCEPT	EXPECTATION	EXAMPLE
C	Tell time and use units of time	Tell time to the nearest minute	Problem: Display clocks with various times and ask students to give the correct time to the nearest minute. Answer: Answers will vary depending on the time of day the teacher asks students to provide the time.

		Problem: The teacher asked the class to look at the clock and tell him the time. Sid said the time was 10:49 and Sal said it was 11 minutes before 11. Who was right? Answer: They are both right. 10:49 and11 minutes before 11 are the same times.

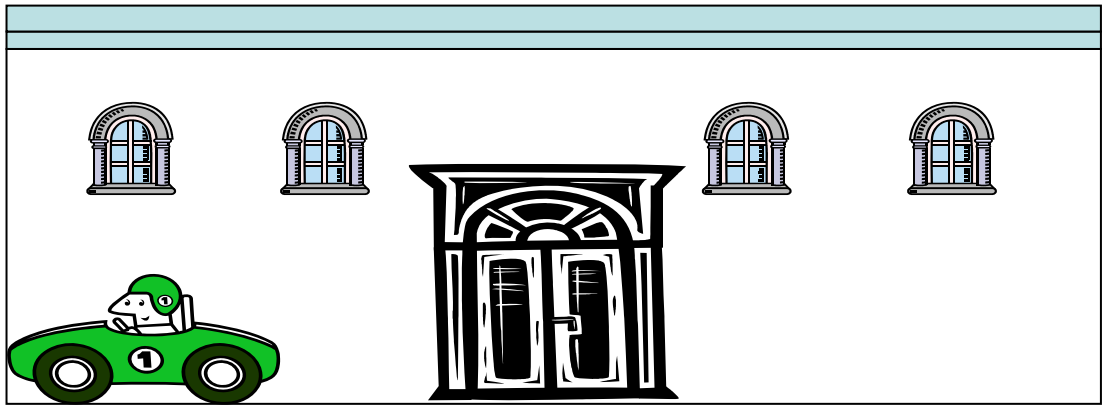
DEFINITIONS:

	CONCEPT	EXPECTATION	EXAMPLE
D	Count and compute money	Determine change from $10.00 and add and subtract money values to $10.00	Problem: Tony has $10.00 to spend at the summer camp carnival. Since the money raised by the carnival will go for a good cause, Tony has decided to spend the entire $10.00. The prices for the carnival activities are shown below and there is no limit on the number of times you can do each activity. Show how Tony could spend the entire $10.00 at the carnival. 

		Answer: Numerous answers are possible. 2 fish ponds ($4) and 2 fun houses ($6) = $10.00; 3 fun houses ($9) and 1ring toss ($1) = $10; 1 fish pond ($2), 1fun house ($3), and 2 face paintings ($5)= $10, etc. Problem: Use carnival activity prices above. Ann bought a ticket for the fish pond and face painting. How much money did she have left from the $10.00 she had to spend? a. $5.00 b. $5.50 c. $6.50 d. $14.50 Answer: b. $5.50

BIG IDEA (2): Apply techniques, tools, and formulas to determine measurements

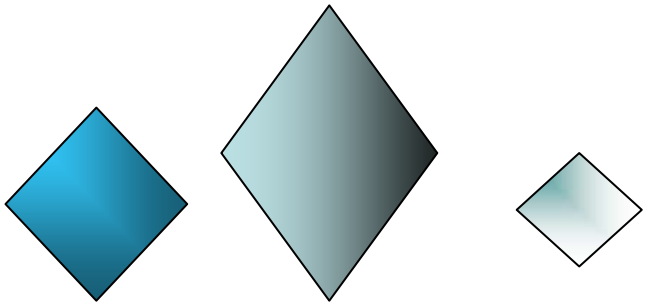
	CONCEPT	EXPECTATION	EXAMPLE
A	Use standard and non-standard measurement	Select and use benchmarks to estimate measurements (linear, capacity, and weight)	Problem: The car is 3 meters long. About how long is the building?



a. 8 meters
b. 10 meters
c. 12 meters
d. 14 meters

Answer:
12 meters

Problem:
Mr. Numero plans to tile his kitchen floor with one of the tiles below.



Which tile would he need the least of to cover his floor completely. Explain why.

Answer:
Tile B. It is the biggest of the three tiles so it would take less of it to cover the floors than the others.

TEACHER NOTES:
"Students should develop mental images or benchmarks that allow them to compare measurements in two systems. Although students at this level do not need to make precise conversions between customary and metric measurements, they should form ideas about relationships between units in the two systems, such as that one centimeter is a little shorter than an inch, that one liter is a little more than a quart, and that one kilogram is a little more than two pounds."[1]
In the customary measurement system, it has been the practice to separate instruction of solid and liquid measure. Osborne (1976, p. 28) attributes this

[1] *Navigating through measurement in grades 3 -5* (p. 3.) (2003). Reston, VA: National Council of Teachers of Mathematics

tradition to the nature of the customary measurement system with quarts, pints, and gallons appearing so different from cubic inches, feet, and yards—a situation that does not exist in the metric system. It is important, however, for the teacher to recognize that measurements of capacity are measurements of volume.

If students have an understanding of the prefixes used in the metric system, they will have less difficulty remembering the values of the more common measures: milli- (thousandth), centi- (hundredth), deci- (tenth), and kilo- (thousand). In linear measure, students should develop benchmark estimates for inches, feet, yards, centimeters, decimeters, and meters. For customary measures of weight, children should have benchmarks for ounces and pounds, and for metric measures of weight, milligrams and grams. For customary measures of capacity students should identify benchmarks for pints, quarts, and gallons, and for metric measures of capacity (liquid measure), milliliters and liters.

Using charts and organizers such as the two below, serve as visual benchmark tools.

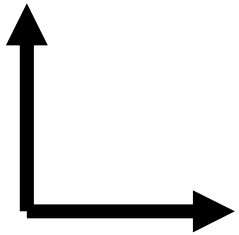
Fractions

1															
1/2								1/2							
¼				¼				¼				1/4			
1/8		1/8		1/8		1/8		1/8		1/8		1/8		1/8	
1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16

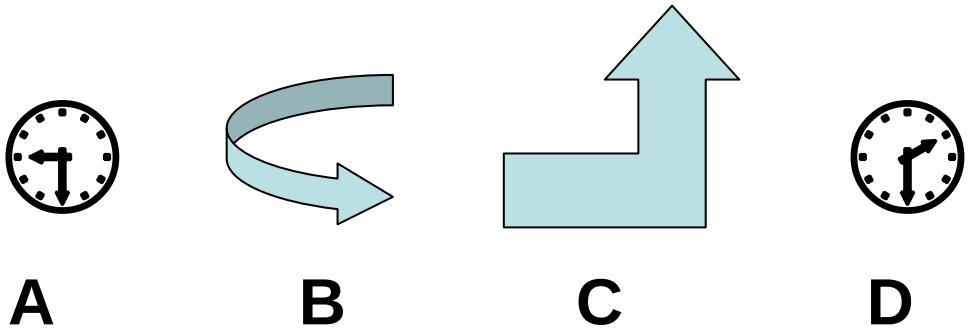
Liquid Capacity

Gallon															
Half-gallon								Half-gallon							
Quart				Quart				Quart				Quart			
Pint		Pint		Pint		Pint		Pint		Pint		Pint		Pint	
C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C

DEFINITIONS:
Benchmark—a reference that is based on situations that are commonly known such as a dollar bill (six inches), the distance of a doorknob from the floor (about a meter or yard), a half-gallon of milk, a two-liter soda, and five pounds of sugar.[2]

	CONCEPT	EXPECTATION	EXAMPLE
B	Use angle measurement	Select and use benchmarks to estimate measurements of 0-, 45-, 90- degree angles	Problem: This angle measures 90^0. 

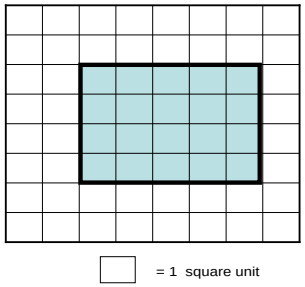
[2] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 174). Reston, VA: Author.

		Which of the following show a 90^0 angle?  **A** **B** **C** **D** Answer: C

DEFINITIONS:
Benchmark—a reference that is based on situations that are commonly known such as a dollar bill (six inches), the distance of a doorknob from the floor (about a meter or yard), a half-gallon of milk, a two-liter soda, and five pounds of sugar.[3]

CONCEPT	EXPECTATION	EXAMPLE
C Apply geometric measurements	Determine the area of a polygon on a rectangular grid	Problem: What is the area of the rectangle on the grid below?

[3] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 174). Reston, VA: Author.



a. 9 square units
b. 10 square units
c. 18 square units
d. 20 square units

Answer:
d. 20 square units

DEFINITIONS:

MEASUREMENT

Grade 5

BIG IDEA (1): Understand measurable attributes of objects and the units, systems and processes of measurement

	CONCEPT	EXPECTATION	
A	Determine unit of measurement	Identify and justify the unit of measure for area (customary and metric)	Problem: A flower bed measures 4 meters by 8 meters. How would you label the area of the flower bed? a. 32 meters b. 32 square meters c. 32 cubic meters d. 32 kilometers Answer: b. 32 square meters Problem: Steve found the area of his basketball court and is trying to remember how to label his answer. Which of the following is the correct unit label? a. feet b. unit feet c. square feet d. cubic feet Answer: c. square feet Problem: Gary and Shirley have been discussing how to label the answer they got for

		the area of the garage floor. Gary says it should be labeled square feet and Shirley says it should be labeled feet. Explain which one is correct and why. Answer: Gary is correct as it should be labeled square feet. Whenever you find area you are multiply units x units which results in units being used as a factor twice or units squared. <u>TEACHER NOTES:</u> Students should recognize the need to select units appropriate to the attribute being measured. Different kinds of units are needed for measuring area than for measuring length. In addition, they should clearly understand the essential importance to give the unit as well as the numerical value when reporting measurements. By definition, area is the number of square units that cover a surface. Thus, answers for area should be labeled in square units.

DEFINITIONS:

	CONCEPT	EXPECTATION	
B	Identify equivalent measures	Identify the equivalent weights and equivalent capacities within a system of measurement	Problem: A measurement of 25 kilograms is the same as: a. 250 grams b. 250 grams c. 2,500 grams d. 250, 000 grams

		Answer: d. 25, 000 grams Problem: The class had 32 quarts of punch for their party. How many gallons of punch did they have? a. 8 gallons b. 64 gallons c. 128 gallons d. 256 gallons Answer: a. 8 gallons <u>TEACHER NOTES:</u> "They should develop an understanding of relationships within each system of measurement (such as twelve inches equals one foot or that one gallon is equivalent to four quarts. In addition, they should learn that units within the metric system are related by factors of ten (e.g., one centimeter equals ten millimeters, and one meter equals one hundred centimeters or one thousand millimeters)."[1]

DEFINITIONS:

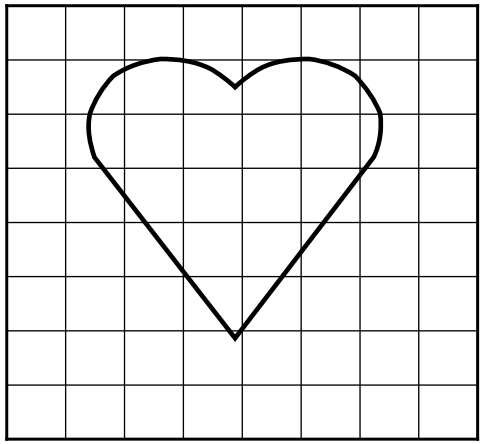
CONCEPT	EXPECTATION	EXAMPLE

[1] *Navigating through measurement in grades 3 -5* (p. 3.) (2003). Reston, VA: National Council of Teachers of Mathematics

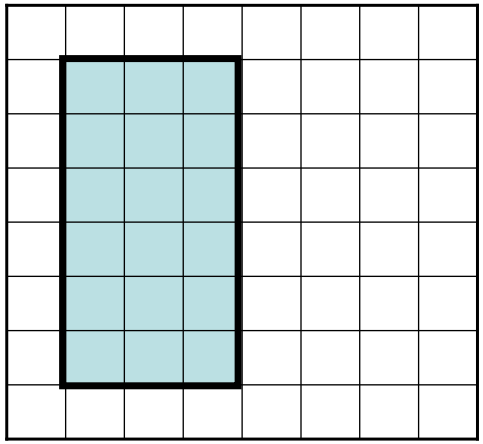
C	Tell time and use units of time	Solve problems involving elapsed time (hours)	Problem: The clock on the left shows the time Emily began her homework. The clock on the right shows the time she ended her homework. How long did she work on her homework?  Answer: 1 hour.

DEFINITIONS:

BIG IDEA (2): Apply techniques, tools, and formulas to determine measurements

CONCEPT	EXPECTATION	EXAMPLE
C Apply geometric measurements	Describe how to solve problems involving area of polygons and non-polygonal regions imposed on a rectangular grid	Problem: Describe how to find the area of the heart on the grid below. Explain how you got your answer.  = 1 square unit Answer: Acceptable answer is about 16 square units. I counted the whole squares inside the heart first, then I put together part squares to make whole squares and added them to my whole square total. Problem:

Find the area of the rectangle on the grid below. Explain how you got your answer.



= 1 square unit

Answer:
I counted and got 18 square units for the area or I multiplied 3 x 6 to get 18 square units.

TEACHER NOTES:
As students have opportunities to look for patterns in the results of their measurements, they recognize that their methods for measuring area can be generalized as formulas. They realize that counting all squares is not necessary in rectangles once the length and width are determined.[2]

[2] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 172). Reston, VA: Author.

DEFINITIONS:

	CONCEPT	EXPECTATION	EXAMPLE
E	Use relationships within a measurement system	Convert from one unit to another within a system of measurement (linear)	Problem: A measurement of 84 inches is how many feet? Show or tell how you got your answer. Answer: 7 feet; There are 12 inches in a foot and 7 x 12 = 84; I added 12 + 12 + 12 + 12 + 12 + 12 + 12; There are 12 inches in a foot so I divided 84 by 12 to get 7; or other valid response. Problem: A measurement of 5 yards is how many inches? a. 1.5 inches b. 15 inches c. 60 inches d. 180 inches Answer: d. 180 inches Problem: Alan has 40, 000 millimeters of kite string. How many meters of kite string does he have? Show how you got your answer. Answer: 1 meter = 1000 millimeters so I took 40, 000 and divided by 1000 to get 40 meters.

		Problem: A measurement of 6 kilometers is how many meters? a. 60,000 meters b. 6,000 meters c. 600 meters d. 60 meters Answer: b. 6,000 meters <u>TEACHER NOTES:</u> "Students should gain facility in expressing measurements in equivalent forms. They use their knowledge of relationships between units and their understanding of multiplicative situations to make conversions such as expressing150 centimeters as 1.5 meters or 3 feet as 36 inches."[3]

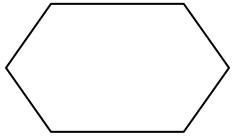
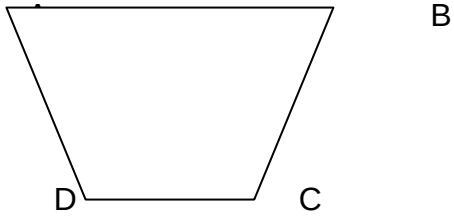
DEFINITIONS:

[3] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 174). Reston, VA: Author.

MEASUREMENT

Grade 6

BIG IDEA (1): Understand measurable attributes of objects and the units, systems and processes of measurement

	CONCEPT	EXPECTATION	
A	Determine unit of measurement	Identify and justify an angle as acute, obtuse, straight or right	Problem: What kinds of angles are in this hexagon- Acute, obtuse, right or straight angles?  Answer: Obtuse Problem: In the quadrilateral below, identify each angle as acute, obtuse or right. Explain your reasoning for each angle.  Answer: Angles A and B are both acute angles because they each measure less than 90º. Angles D and C are both obtuse angles because they each measure more than 90º.

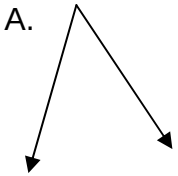
Problem:
Explain why half of an obtuse angle can't be an obtuse angle?

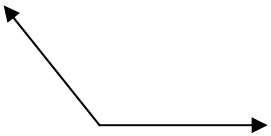
Answer:
A straight angle measures 180º. Half of a straight angle would be a 90º angle. All obtuse angles are less than 180º but more than 90º, so if I take half of something that is less than 180º, then the answer will be less than 90º. Therefore, the answer cannot be obtuse.

Problem:
When the clock face displays 2 o'clock, what kind of angle do the two hands form? Explain your answer

Answer:
An acute angle because it measures less than 90 degrees.

Problem:
Identify the following angles as acute, obtuse, right, or straight and then justify your result

A. 

B. 

Answer:
A is acute because it measures less than 90 degrees.
B is obtuse because it measures more than 90 degrees.
<u>TEACHER NOTES:</u>
Developing and using measurement concepts and skills should be ongoing

		throughout the school year rather than treated as a separate unit of study. Many measurement topics are closely related to other mathematics content and strands. Middle grade students should become proficient at measuring angles and using ratio and proportion to solve problems involving scaling, similarity, and derived measures.

	CONCEPT	EXPECTATION	EXAMPLE
C	Tell time and use units of time	Solve problems involving elapse time (hours and minutes)	Jim arrived at school at 7:45 am and left school at 3:10 pm. How long was he at school? Answer: 7hour 25 minutes Problem: Lucy and Max went to the theater to watch two different movies. The first movie got out at 2:47 p.m. and the second movie started at 3:38 p.m.. They planned to play video games after the first movie ended until the second one began. How much time did they have to play video games? Answer: 51 minutes Problem: John's favorite reality show comes on every Tuesday at 7:30 pm. It is on for

		1 and 1/2 hours. John must be in bed by 9:15 pm. Will John be able to see the end of his favorite show? Explain your answer. Answer: Yes, because the show ends at 9:00 p.m. Problem: Find the length of your airplane flight if the following schedule is shown on your ticket. Leave City A 1:23 p.m. Arrive City B 3:05 p.m. Explain how you got your answer. Answer: Example: From 1:23 to 3:00 is 1 hr 37 minutes. Add 5 more minutes to get 1 hr and 42 minutes.

BIG IDEA (2): Apply techniques, tools, and formulas to determine measurements

	CONCEPT	EXPECTATION	EXAMPLE
A	Use standard and non-standard unit of measurement	Estimate a measurement using either <u>standard</u> or <u>non-standard</u> unit of measurement	Problem: _____ The length of this line segment is about a. 25 mm b. 1 cm c. 50 mm

d. 6 cm

Answer:
6 cm

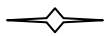
Problem:
In England and America, it's still common to measure the height of a horse in "hands." If a hand is equal to the width of your palm, estimate your own height in hands.

Answer**:**
Answers will vary. A hand is approximately 4 inches. Five feet would be about 15 hands.

Problem:
John wants to know the distance from the floor to the top of fourth book shelf in his grandfather's office. He used a regular sheet of 8 1/2 "x 11" of paper. He estimated the height to be 4 to 5 sheets. Do you agree? Explain

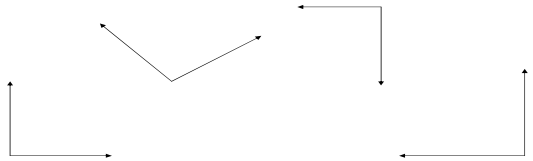
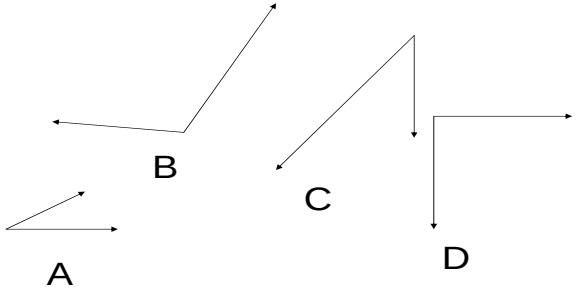
Answer:
I agree. Depending on how he turns the paper to measure it, it would be between 34 and 44 inches, which is a reasonable measurement from the floor to the top of the fourth book shelf.

Problem:
Use the following shape as your unit of measure :

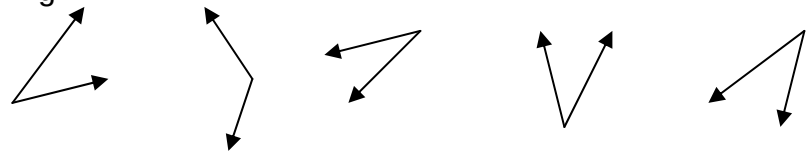
Estimate how many  it would take to find the length of the following line:

		Answer: About 7; 7 and ½ or 8 would be acceptable. <u>TEACHER NOTES:</u> "When students measure an object, the result should make sense; estimates and benchmarks can help students recognize when a measurement is reasonable. Students can use their sense of common units to estimate measurements; for example, the height of a classroom door is about two meters, it takes about ten minutes to walk from the middle school to the high school, or the textbook weighs about two pounds. They should also be able to estimate large measurements; for instance, the distance between the middle school and the high school is about the length of ten football fields."[1]

[1] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* (p. 243). Reston, VA: NCTM.

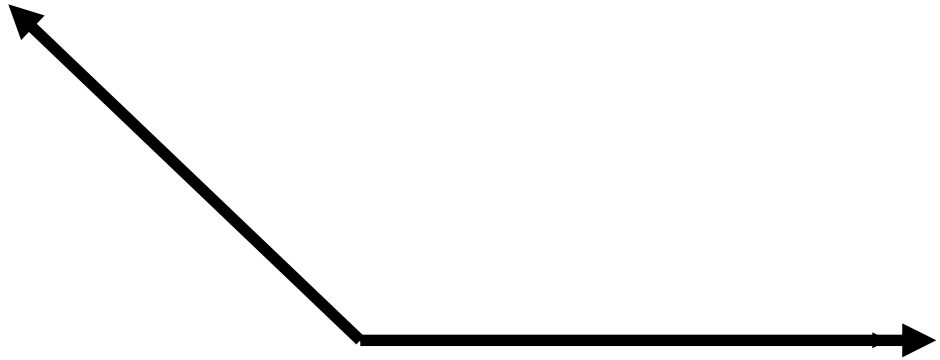
CONCEPT	EXPECTATION	EXAMPLE
B Use angle measurement	Select and use benchmarks to estimate measurements of 0-, 45-, 90-,180-, 360- degree angles	Problem: These angles measure 90^0 and are referred to as right angle.  Which of the following are angles are less than 90^0 and which are greater than 90^0?  Answer: A and C are less than 90^0; B is greater than 90^0.

Problem:
Circle the angle below that does not belong with the other angles. Explain your reasoning.

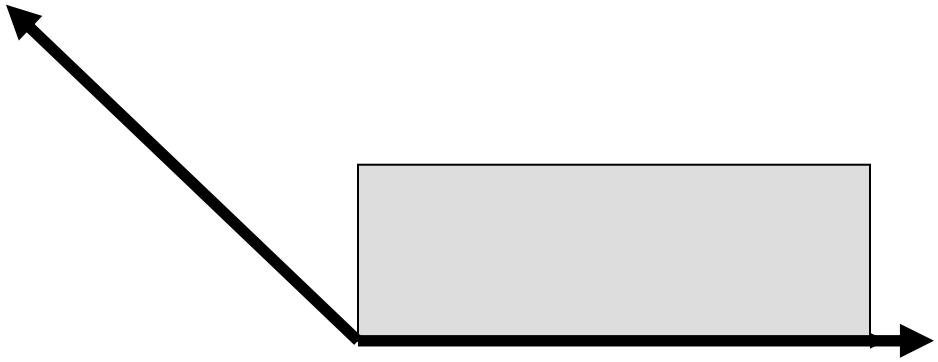


Answer:
Angle B does not belong. All the other angles are acute angles, but angle B is obtuse.

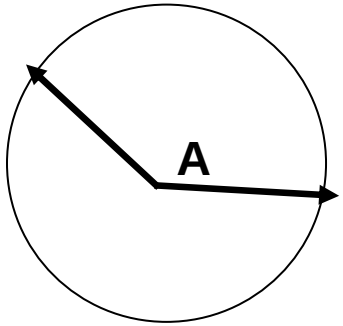
Problem:
Explain how a 5" X 3" index card can be used as a benchmark to determine if the angle below is acute, right, or obtuse.



Answer:
You could line the card up to show where a right angle would fall within the given angle and see that this angle rotates more than or measures more than 90^0.



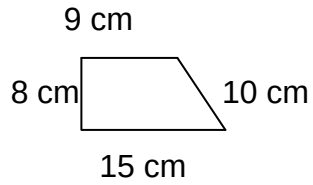
Problem:
Which of the following is the best estimate for angle A? Explain your answer.



a. 20° b. 80° c. 120° d. 175°

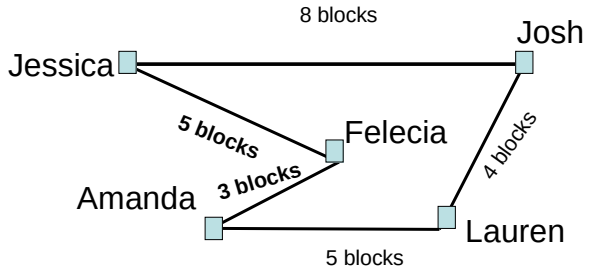
Answer:
C. 120° . Using benchmark angles such as 90 degrees and 180 degrees, we see that the angle is closer to 120 degrees

DEFINITIONS:
Benchmark—a reference that is based on situations that are commonly known such as a dollar bill (six inches), the distance of a doorknob from the floor (about a meter or yard), a half-gallon of milk, a two-liter soda, and five pounds of sugar.[2]

	CONCEPT	EXPECTATION	EXAMPLE
C	Apply geometric measurements	Describe how to solve problems involving area and perimeter of polygons	Problem:  Explain how to find the area of this trapezoid with two right angles. Answer: You find the mean of the two bases and multiply by the height. (9 + 15)/2 x 8 = 96 cm^2 Problem: One day, Amanda rode her bike to visit some of her friends. First, she went to Lauren's house, then to Josh's house, then to Jessica's house, then to

[2] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 174). Reston, VA: Author.

Felicia's house, and finally back to her own house. How many blocks did Amanda travel that day? Explain how you found your answer and identify whether this is an area or perimeter problem.



Not to scale.

Answer:
5 + 4 + 8+ 5 + 3= 25 blocks
This is a perimeter problem because you are measuring a distance around the shape.

Problem:
If the area of a rectangular field is 36 square yards, what could be the perimeter of the field? Explain your answer.

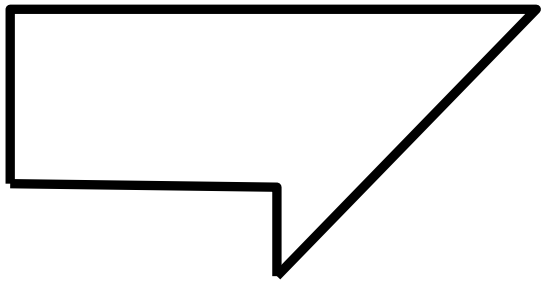
[3] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* (p. 243). Reston, VA: NCTM.

Answer:
Possible perimeters include the following:

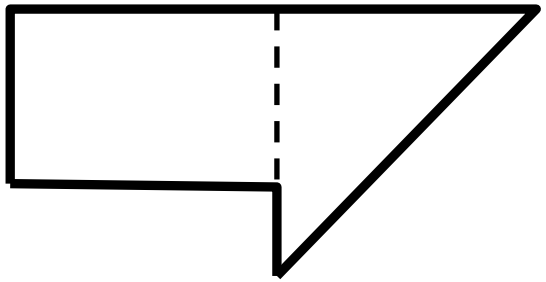
Length	Width	Perimeter
1	36	74
2	18	40
3	12	30
4	9	26
6	6	24
9	4	26
12	3	30
18	2	40
36	1	74

Answer should include how the perimeter is determined by 2(l w) or 2(l)+2(w) or l + l + w + w.

While drawing the rooms in the school and finding their area, your teacher came into an unusual room and hasn't any idea on how she would find the area. Please help her to make sense of the room by explaining how you could find the area of the room.



Answer:
You can take the unusual shape and draw in auxiliary lines or helping lines in order to make it into shapes that are more familiar. Then find the area of each familiar shape and add them together to get the total area.



The lines in this shape separate it into a rectangle and triangle. Find the areas of rectangle and triangle and add them together which results in the area of the entire shape.

TEACHER NOTES:
"Students should become proficient in composing and decomposing two- and three-dimensional shapes in order to find the lengths, areas, and volumes of various complex objects."[3]

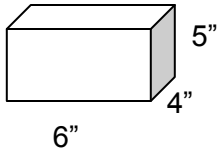
DEFINITIONS:

CONCEPT	EXPECTATION	EXAMPLE
E Use relationships within a measurement system	Convert from one unit to another within a system of measurement (mass and weight)	Problem: Mary's dog weighs 21 kg. She says this is the same as 2100 g while Dan says 21 kg equals 21,000 g. Who is correct and why? Answer: Dan is. There are 1000 g in a kg and 21 x 1000 = 21,000 kg. Problem: 1 Troy pound = 12 Troy ounces 1 Troy ounce = 20 pennyweights 1 pennyweight = 24 grams Use the information above to place the following weights in order from smallest to largest. 32 Troy ounces 60 pennyweights 120 grams 2 Troy pounds Answer: 2 Troy Pounds 32 Troy ounces 60 pennyweights 120 grams Problem: Determine the number of pounds contained in a box labeled as having 128 ounces. Show how you arrived at your answer. Answer: 128 / 16 = 8 pounds; 128/x =16/1

MEASUREMENT

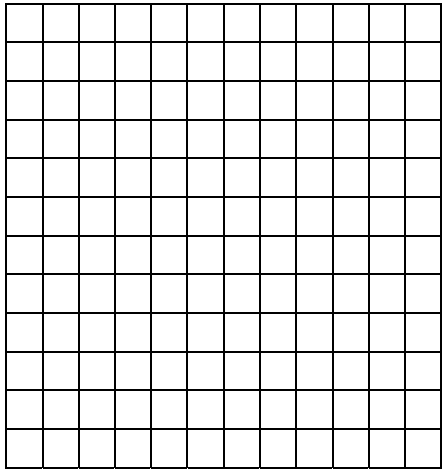
Grade 7

BIG IDEA (1): Understand measurable objects and the units, systems and processes of measurement.

	CONCEPT	EXPECTATION	EXAMPLE
A	Determine unit of measurement	Identify and justify the unit of measure for volume (customary and metric)	Problem: Mary said the volume of this prism is 120 square inches while Sam said it is 120 cubic inches.  Who is correct? Explain your thinking. Answer: Sam is correct. The volume is found by multiplying 6” ● 5” ● 4”. Since three units are being multiplied, the volume is expressed in cubic units. The correct volume is 120 cubic inches.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Tasha is on the refreshment committee for the spring dance at her school. The committee plans to serve punch and Tasha is in charge of purchasing the fruit juice that will be used to create the punch. If the committee wants to make 200 servings of punch, which of the following is the most likely amount of fruit juice needed? A. 30 milliliters B. 30 ceniliters C. 30 kL Answer: 30 liters Problem: Name a customary and metric unit that could be used to measure the following items. For example, a trip from St. Louis to Kansas City could be measured in miles or kilometers. 1. Volume of soda in a can 2. Volume of water used in a school each day Possible answers: 1. The soda could be measured in ounces or milliliters. 2. The water could be measured in gallons or liters.

CONCEPT	EXPECTATI	EXAMPLE
Identify equivalent measures	Identify the equivalent area measures within a system of measurement	Problem: The area of the rectangle pictured below is 8 square meters.  Express the area in square centimeters. Provide the work that supports your answer. Answer: 80,000 cm^2 1 m = 100 cm so 1 m^2 = $(100\ cm)^2$ = 10,000 cm^2. This means that 8 m^2 is equivalent to 80,000 cm^2. Or 80,000 cm^2 The dimensions of the rectangle can be converted to 400 cm by 200 cm. So the area is found by multiplying 400 cm ● 200 cm. The area is 80,000 cm^2.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: How many square inches are in a square foot? Explain your reasoning. Answer: There are 144 square inches in a square foot. Each foot is equivalent to 12 inches so a square foot is 12 inches by 12 inches or 144 square inches. Or This picture is a square foot. I divided each side into 12 inches. There are 144 square inches in one square foot. 

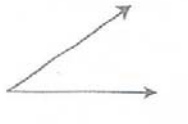
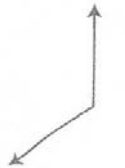
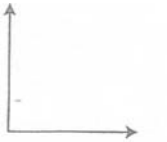
CONCEPT	EXPECTATION	EXAMPLE
		Problem: Kelly's family is moving to a new house. Since Kelly is the oldest child in her family, she will have her choice of bedrooms. She decides to compare the area of the floors in each bedroom to determine which is the largest. The floor area of one bedroom is 90 square feet. The floor area of the other is 16 square yards. Which room is the largest? Provide the work that supports your answer. Answer: The room with the floor area of 16 square yards is the largest. Since there are 9 square feet in each square yard, the area of the second room could be converted to square feet by multiplying 16 by 9. This means the area of this room could be expressed as 144 square feet which is greater than the 90 square feet area of the other room. Or The room with the floor area of 16 square yards is the largest. Since there are 9 square feet in each square yard, the area of the first room could be converted to square yards by dividing 90 by 9. This means the area of this room could be expressed as 10 square yards which is less than the 16 square yard area of the other room.

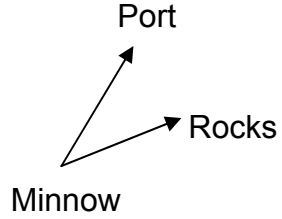
CONCEPT	EXPECTATION	EXAMPLE
		Problem: Sue's parents are buying new carpet for her bedroom. The room measures 9 feet by 12 feet. Carpet is sold in square yards. How much carpet will be needed for Sue's room? Explain how you arrived at your answer. Answer: 12 square yards The amount of carpet needed is found by calculating the area of the floor. The area of the room is 108 square feet. Since there are 9 square feet in a square yard, dividing 108 by 9 results in 12 square yards of carpet. Or 12 square yards The dimensions of the room can be converted into 3 yards by 4 yards. The amount of carpet needed is found by calculating the area of the floor. The area of the room would b e 12 square yards. This is the amount of carpet needed.

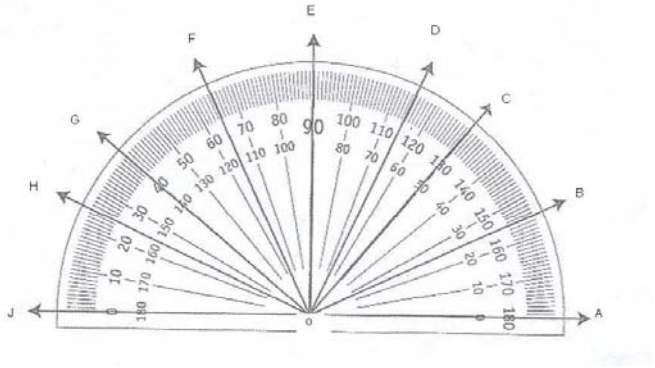
	CONCEPT	EXPECTATION	EXAMPLE
C	Tell and use units of time	Solve problems involving addition and subtraction of time (hours, minutes, and seconds)	Problem: Janie competed in a 30 km race. She ran the first 15 km in 1 hour, 27 minutes, 35 seconds. Her time for the second 15 km was 1 hour, 49 minutes, 52 seconds. What was her total time for the 30 km? Provide the work that supports your answer. Answer: 3 hr. 17min. 27sec. 1 h 27 m 35 s + 1 h 49 m 52 s 2 h 76 m 87 s = 2 h 77 m 27 s = 3 hr. 17 min. 27 sec. Problem: Tim knows that to drive to his uncle's house it takes 2 hours, 47 minutes, and 39 seconds. He also knows that to drive to his grandparents' house it takes 5 hours, 25 minutes, and 17 seconds. How much longer does it take to get to his grandparents' house than to his uncle's house? Provide the work that supports your answer. Answer: 2 hours, 37 minutes, 38 seconds 5 hrs. 25 min. 17 sec. - 2 hrs. 47 min. 39 sec. 4 hours 84 min 77 sec – 2 hours 47 min 39 sec. 2 hours 37 min 38 seconds.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Korin is going to a movie which begins at 1:00 p.m. • It takes her 15 minutes, 45 seconds to walk to the bus. • The bus ride takes 38 minutes, 10 seconds. • It takes her 5 minutes, 35 seconds to walk from the bus to the movie. What time should she leave her house if she wants to arrive at the movie theater 10 minutes before the movie starts? Answer: She should leave her house at 11:50:30 a.m. Problem: Zach starts mowing lawns at 10:30:15 a.m. Saturday morning. He finishes mowing the last lawn at 3:20:12 p.m. Saturday afternoon. How long did he spend mowing? Answer: 4 hours, 39 minutes, 57 seconds

BIG IDEA (2): Apply appropriate techniques, tools, and formulas to determine measurements.

	CONCEPT	EXPECTATION	EXAMPLE
A	Use angle measurement	Use tools to measure angles to the nearest degree	Problem: Use a protractor to measure each angle to the nearest degree. 1.  2.  3.  Answer: 1. Approximately 35 degrees. 2. Approximately 125 degrees. 3. Approximately 90 degrees.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: During a stormy night at sea, the Coast Guard had to warn a small boat, the U.S.S. Minnow to steer to the left to avoid the rocks and make it safely to the port. If the Minnow is headed straight for the rocks, how many degrees must the coast guard tell the small boat to turn in order to make it safely to the port? Use your protractor and measure to the nearest degree.  Answer: The Minnow would need to turn approximately 37 degrees.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Haylee believes that angle HOJ, pictured below, measures 155 degrees. Is Haylee  Answer: Haylee is not correct. Angle HOJ is an acute angle so its measure must be less than 90 degrees. Haylee should use the outer scale of the protractor since one ray of angle HOJ lines up with the 0 on the left side of the protractor. This means that the measure of angle HOJ is approximately 25 degrees.

CONCEPT	EXPECTATION	EXAMPLE

CONCEPT	EXPECTATION	EXAMPLE
C Apply geometric measurements	Describe how to solve problems involving circumference and/or area of a circle	Problem: A bicycle wheel is 26 inches in diameter. 1. Describe how you could use either the radius or the diameter to find the circumference. 2. What is the circumference of the wheel? Provide the work that supports your answer. Answer: 1. If I used the diameter, I would need to multiply the diameter by a value for pi. If I used the radius, I would first multiply the radius by 2 and then multiply by a value for pi. 2. Approximately 81.6 inches C = d ●pi C≈ 26 ● 3.14 C≈ 81.6 inches Problem: The spray from a lawn sprinkler makes a circle with a radius of 40 feet. What is the area of the circle of lawn watered by the sprinkler? Provide the work that supports your answer. Answer: 5,024 square feet $A = r^2$ ● pi $A \approx 40^2$ ● 3.14 A ≈ 5024 square feet

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Mr. Smith owns a Merry-Go-Round with a circular floor whose radius is 10 feet. He plans to paint the floor and needs to know the area so he can purchase the correct amount of paint. He also wants to put a decorative strip around the edge of the floor going all around the circumference of the circle. 1. What is the area of the floor? 2. How many feet of decorative strip will be needed? Provide the work that supports your answers. Answer: 1. Approximately 314 square feet $A = r^2 \bullet pi$ $A \approx 10^2 \bullet 3.14$ $A \approx 314$ square feet 2. Approximately 63 feet of decorative strip $C = d \bullet pi$ $C \approx 20 \bullet 3.14$ $C \approx 63$ feet

CONCEPT	EXPECTATION	EXAMPLE
		Problem: A giant sequoia tree has a circumference of 75 feet. What is the diameter of the tree? Provide the work that supports your answer. Answer: The diameter is approximately 23.9 feet. The diameter can be found by dividing the circumference by pi. $d \approx 75 \div 3.14$ $d \approx 23.9$ feet Problem: Carla's class is measuring the circumference and diameter of several different circles. Carla reports that the circle she measured has a circumference of 40 inches and a diameter of 7.5 inches. Explain how Carla's teacher can tell immediately that Carla has measured incorrectly. Answer: The ratio of the circumference to the diameter of a circle is pi. So when the circumference value is divided by the diameter value the quotient should be approximately 3.14. In this case, 40 divided by 7.5 is approximately 5.3. Since this is considerably larger than 3.14, Carla measured incorrectly.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Two circles have radii of 5 cm and 10 cm. How do the areas of the two circles compare? Explain your thinking. Answer: The circle with a radius of 10 cm has an area that is four times the area of the circle with a radius of 5 cm. The area of the circle with a radius of 5 cm is approximately 78.5 square cm. The area of the larger circle is approximately 314 square meters. The area of the larger circle is four times larger than the area of the smaller circle. Or The area increases by the square of the scale factor used. Since the larger circle has a diameter that is twice as large as the smaller circle, the area will be 2^2 or 4 times larger.

	CONCEPT	EXPECTATION	EXAMPLE
D	Analyze precision	Analyze **precision** and accuracy in measurement situations	Problem: Bob measured the length of the room and found it was 11.8 meters. Julie measured it and found it to be 11.76 meters. Which measurement is the most precise? Explain your thinking. Answer: Julie's measurement of 11.76 is most precise because she reported her answer to the nearest hundredth of a meter. A hundredth of a meter is a smaller unit of measure than a tenth of a meter which is how Bob reported his measurement. Problem: Which of the following measurements is the more precise? Explain your thinking. A. 5 g B. 8 mg C. 3 kg Answer: B. Since a milligram is a smaller unit of measure than a gram or kilogram, 8 mg is the more precise measurement.

DEFINITIONS:

precision – an indication of how finely a measurement was made. When you calculate with measured valued, you may need to round to the smallest place in the roughest actual measurements.[1]

[1] Geometry to go (p.499). (2001). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Your friend says that 3.653 km is more precise than 4.8 cm because a thousandths unit is a smaller unit than a tenths unit. Explain to your friend why her reasoning is incorrect. Answer: Since the two measurements are expressed in different units, you cannot look only at the place values of the measurements. Centimeters are a smaller unit of measure than kilometers so 4.8 centimeters is the more precise measurement. Problem: 1. Add the lengths 6.45 meters, 5.712 meters, and 3.1 meters. 2. To what place, should the sum be rounded? Explain your thinking. Answer: 1. 6.45 + 5.712 + 3.1 = 15.262 2. The least precise measurement is 3.1 meters. The sum should be rounded to the nearest tenth of a meter (15.3) which is the unit of the least precise measure.

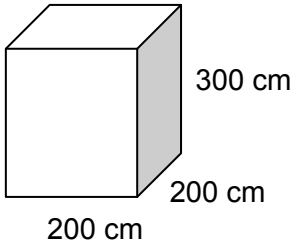
CONCEPT	EXPECTATION	EXAMPLE
E Use relationships within a measurement system	Convert from one unit to another within a system of measurement (capacity)	Problem: Janet is making punch to serve the children attending a summer camp. She has made five gallons of punch. The punch will be served in 8 ounce cups. How many cups of punch will she be able to serve? Answer: Each gallon has 4 quarts Each quart has 2 pints Each pint has 2 cups 2 x 2 x 4 = 16 cups in each gallon Since 8 ounces equals 1 cup, each gallon has 16 cups. Therefore, five gallons of punch will serve 80 children. Problem: The average flow of water over Niagara Falls is 6,008,835,000 mL per second. How many liters per second is this? Answer: 6,008,835 liters To convert milliliters to liters, divide by 1000. 6,008,835,000 ÷ 1000= 6,008,835 liters.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: At Store A, a 2 ¼ quart container of salsa costs $7.85. Store B charges the same price for a container which holds 10 cups of the same salsa. Which store gives you the most salsa for your money? Explain your thinking. Answer: Store B. Since there are 2 cups in a pint and 2 pints in a quart, there are 4 cups in a quart. Store A sells salsa in 2 ¼ quart containers. This means there are 9 cups in the container. Since Store B sells 10 cups for the same price, Store B has the better buy.

MEASUREMENT

Grade 8

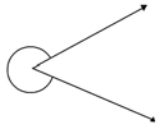
BIG IDEA (1): Understand measurable attributes of objects and the units, systems, and processes of measurement.

	CONCEPT	EXPECTATION	EXAMPLE
B	Identify equivalent measures	Identify the equivalent volume measures within a system of measurement (e.g. m^3 to cm^3)	Problem: The rectangular prism below has a volume of 12,000,000 cm^3.  Express the volume in cubic meters. Answer: The dimensions of the cube may be converted to 2 m, 2 m, and 3 m. This means the volume will be 12 cubic meters. Or One cubic meter is equivalent to 1,000,000 cubic centimeters. The volume of 12,000,000 cubic centimeters would be equivalent to 12 cubic meters.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: A swimming pool has dimensions 12 feet by 15 feet by 6 feet. 1) Find the volume of the pool in cubic feet. 2) Find the volume of the pool in cubic yards. Explain your thinking. Answer: 1) The volume is found by multiplying 12 feet ● 15 feet ● 6 feet. The volume is 1080 cubic feet. 2) 40 cubic yards. There are 9 cubic feet in one cubic yard because there are three feet in a yard and 3 feet ● 3 feet ● 3 feet is equal to 27 cubic feet. So the volume of the pool in cubic yards can be found by dividing 1080 by 27. Or The dimensions of the pool can be changed to 4 yards by 5 yards by 2 yards. To find the volume you can multiply 4 yards ● 5 yards ● 2 yards. This means the volume is 40 cubic yards.

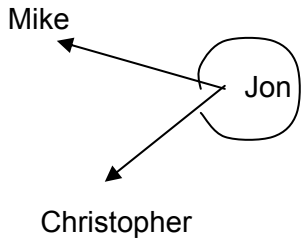
CONCEPT	EXPECTATION	EXAMPLE
		Problem: The volume of a marble is 3.9 cm^3. Express this volume in milliliters. Answer: One milliliter is equivalent to one cubic centimeter. The volume of the marble is 3.9 mL. Problem: The volume of a cube is 10 cubic feet. Express the volume in cubic inches. Provide the work that supports your answer. Answer: 1728 cubic inches 1 foot = 12 inches 1 cubic foot = (12 in)(12 in)(12 in) = 1728 cubic inches 10 ● 1728 = 17,280 cubic inches

BIG IDEA (2): Apply appropriate techniques, tools, and formulas to determine measurements.

	CONCEPT	EXPECTATION	EXAMPLE
B	Use angle measurement	Use tools to determine the measure of **reflex** angles to the nearest degree	Problem: Describe how you could find the measure of the reflex angle indicated by the curved line below.  Sample Answers: A full circle is 360°. I could measure the smaller angle and subtract from 360 to find the measure of the angle indicated by the curved line. Or I could extend one of the rays past the vertex. I would then measure the angle between the other ray and the line I drew. Then I could add 180° to find the total angle.

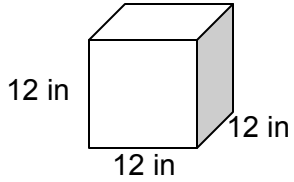
DEFINITIONS:
reflex angles: an angle that measures more than 180°.[1]

[1] Algebra to go (p.502). (2000). Wilmington, MA: Great Source Education Group, Inc.

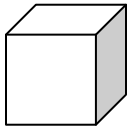
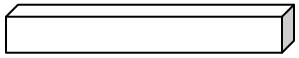
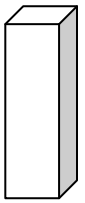
CONCEPT	EXPECTATION	EXAMPLE
		Problem: Jon was skateboarding toward his friend Mike. While on his skateboard, Jon was able to leap in the air and turn to his right to face his friend Christopher. How many degrees did Jon turn on his skateboard to see Christopher? Explain how you arrived at your answer.  Answer: Approximately 305º I measured the angle between Mike and Christopher to be 55°. I subtracted this from 360° which represents Jon turning in a full circle. This means he turned 305°. Or I drew a line from Christopher to past Jon and measured the angle from that line to Mike – 125 degrees. Then I added 180 ° for the part below the line I drew for a total of 305°.

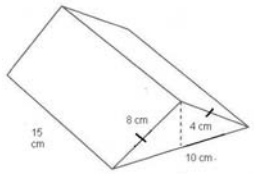
CONCEPT	EXPECTATION	EXAMPLE
		Problem: A clock shows that it is exactly 8:00. Through what angle will the second hand move as it travels from the minute hand to the hour hand? Explain how you arrived at your answer.  Answer: The second hand will travel 180° to get to the 6. It will then travel 60 more degrees to get to the 8. So it travels a total of 240°. Or If the second hand traveled completely around the clock it would move 360°. I divided 360 by 12 to find that each number moved represents 30 degrees. By the time the second hand go to the 8 it would have traveled 30 ● 8 or 240 degrees. Or If the second hand traveled completely around the clock it would move 360°. I divided 360 by 12 to find that each number moved represents 30 degrees. I moved backwards from the 12 to the 8 which is 30 ● 4 or 120 degrees. I subtracted this from 360 and got 240°.

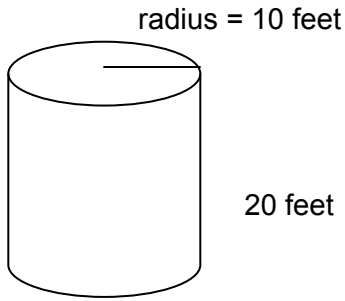
CONCEPT	EXPECTATION	EXAMPLE
C Apply geometric measurements	Describe how to solve problems involving surface area and/or volume of a rectangular or triangular prism or cylinder	Problem: The owner of the diamond store wants to purchase a jeweled box that has a capacity of 60 cubic inches to use to package jewelry. He can use either a 3”x 4”x 5” box or a 2”x 5”x 6” box. One box will be less expensive to make because it has a smaller surface area and will require fewer jewels for decoration. Which of the boxes has the smaller surface area? Provide the work that supports your answer. Answer: The 3” x 4” x 5” box has the smallest surface area. 2(3”x 4”) + 2(4” x 5”) + 2(3” x 5”) 2(12) + 2(20) + 2 (15) 24 + 40 + 30 94 square inches 2(2” x 5”) + 2 (5” x 6”) + 2(2” x 6”) 2(10) + 2(30) + 2(12) 20 + 60 + 24 104 square inches Problem: Give the dimensions of three different rectangular prisms which have a volume of 120 cm^3. Answer: There are many possible correct answers. Some samples are given below. 1 cm x 2 cm x 60 cm 2 cm x 4 cm x 15 cm 1 cm x 12 cm x 10 cm 1 cm x 1 cm x 120 cm 3 cm x 4 cm x 10 cm 3 cm x 8 cm x 5 cm 4 cm x 5 cm x 6 cm 1 cm x 5 cm x 24 cm

CONCEPT	EXPECTATION	EXAMPLE
		Problem: A large cube-shaped box measures 12 inches x 12 inches x 12 inches. How many smaller cube-shaped boxes measuring 3 inches long on each side would fit inside the larger box? Explain your thinking.  Answer: 64 smaller cubes would fit inside the larger box. The volume of the larger box can be found by multiplying 12 ● 12 ●12 which results in a volume of 1728 cubic inches. The smaller boxes have a volume of 27 cubic inches (3 ● 3 ● 3). By dividing 1728 by 27 I found that 64 of the smaller boxes would fit inside. Or Since the smaller boxes are four inches long on each side, 4 of them would fit along a side of the larger cube. I can multiply 4 ● 4 ● 4 to find that 64 of the smaller boxes would fit inside the larger one.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: A company packages its product in large boxes which have a length of 1 foot, a width of 2 feet, and a height of 2 ½ feet. If the company doubles each dimension in order to produce a larger box, how will the volume of the new box compare to the volume of the original box? Explain your thinking. Answer: The volume of the new box will be 8 times that of the original. The volume of the original box is 5 cubic feet (1 ● 2 ● 2 ½). After doubling the dimensions, the volume of the new box will be 40 feet (2 ● 4 ● 5). So the volume of the new box is 8 times the volume of the original. Or Since all three dimensions are doubled, the new box will have a volume that is 2^3 or 8 times larger than the original. This is because the volume increases by the cube of the scale factor.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: A juice company is investigating selling its juice in rectangular boxes. Three rectangular prisms are suggested as possible packages. The prisms all have the same volume but the company wants to choose the package which will be the least expensive to produce. The prisms are shown below. Without knowing the dimensions, how can you tell the prism that will have the smallest surface area and as a result be the least expensive to produce? A .  B.  C.  Answer: If all the prisms have the same volume, Box A will have the least surface area because it is the closest to a cube.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: The base of the prism shown below is an isosceles triangle with two sides which measure 8 units each.  1. Find the surface area of the prism and provide the work that supports your answer. 2. Find the volume of the prism and provide the work that supports your answer. Answer: 1. Surface area: 430 square units Rectangular faces (2)(15)(8) = 240 square units (1)(15)(10) = 150 square units Triangular bases (2)(1/2 ●10 ● 4) = 40 square units Surface area = 240 + 150 + 40 = 430 square units 2. Volume: 300 cubic units Area of triangular base = (1/2)(10)(4) = 20 square units Volume = area of base ● height = 20 ●15 = 300 cubic units

CONCEPT	EXPECTATION	EXAMPLE
		Problem: A cylindrical storage tank has a radius of 10 feet and a height of 20 feet.  1. Find the surface area of the cylinder. Provide the work that supports your answer. 2. Find the volume of the cylinder. Provide the work that supports your answer. Answer: 1. The surface area is approximately 1884 square feet. The area of the circular bases ≈ $(2)(10^2)(3.14)$ ≈ 628 square feet Rectangle ≈ (2)(10)(3.14)(20) ≈ 1256 square feet Total surface area ≈ 628 + 1256 = 1884 square feet 2. The volume is approximately 6280 cubic feet. Volume = Area of Base ● height ≈ $(10^2)(3.14)$ ● 20 ≈ 6280 cubic feet

	CONCEPT	EXPECTATION	EXAMPLE
D	Analyze precision	Analyze **precision** and accuracy in measurement situation and determine number of significant digits	Problem: Order these measurements from least precise to most precise. Explain your thinking. 15.23 grams 6.9 grams 3 kilograms Answer: 3 kilograms, 6.9 grams, 15.23 grams A kilogram is a larger unit than a gram so it would be the least precise. A hundredth of a gram is a smaller unit than a tenth of a gram so 15.23 grams is more precise than 6.9 grams Problem: 1. Find the sum of these lengths: 15.1 m 1.23 m 14.27 m 1.635 m 2. To what place, should the sum be rounded. Explain your thinking. Answer: 1. The sum of the measures above is 32.235 meters. 2. The least precise unit of measure is a tenth of a meter so the answer would be rounded to 32.2 meters.

DEFINITIONS:

precision – an indication of how finely a measurement was made. When you calculate with measured valued, you may need to round to the smallest place in the roughest actual measurements.[2]

[2] Geometry to go (p.499). (2001). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: The length of a pencil is expressed as 0.0907 m. How many significant digits does this measurement have? Explain your thinking. Answer: There are 3 significant digits. The 9 and 7 are significant because nonzero digits are always significant. The zero between them is significant. The other zeros are not significant. Problem: Find the number of significant digits in each of the following measures. 1. 15.20 m 2. 7.006 m 3. 0.0053 m 4. 0.0102 m Answer: 1. 4 significant digits 2. 4 significant digits 3. 2 significant digits 4. 3 significant digits

CONCEPT	EXPECTATION	EXAMPLE
		Teacher Note: The rules for determining significant digits are as follows[3]: • Nonzero digits are always significant. • **For decimal numbers between 0 and 1:** Zeros to the left of ALL the nonzero digits are not significant. All other zeros are significant. For example, given 0.006040: 0.00**6040** The underlined digits are NOT significant. The bold numbers are significant. • For positive integers: Zeros to the right of ALL the nonzero digits are not significant. Zeros between nonzero digits are significant. For example, given 203,400: **203,4**00 The underlined digits are NOT significant. The bold numbers are significant. • For noninteger decimal numbers greater than 1: All zeros are significant. For example, given 350.07050: **350.07050** The bold number are significant.

[3] Pre-Algebra (p. 157). (2001). Upper Saddle River, NJ: Prentice Hall.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: Mr. Jones buys a lot on which to build a new house. The lot measures 165.4 meters by 151 meters. What is the area of the plot? Use significant digits. Explain your thinking. Answer: To find the area of the lot, multiply 165.4 by 161. The area is 26,629.4 square meters. To use significant digits, round the answer to match the least number of significant digits in the problem. Since 165.4 has 4 significant digits and 151 has 3, the answer will be rounded to 3 significant digits. The area would be expressed as 26,600 square meters.

CONCEPT	EXPECTATION	EXAMPLE
E Use relationships within a measurement system	Convert square or cubic units to equivalent square or cubic units within the same system of measurement	Problem: Dexter is planning to install new carpeting in his bedroom. His bedroom measures 10' by 15'. • Store A offers the carpet that Dexter wants for $3.75 per square foot. • Store B offers the same carpet for $25.50 per square yard. 1. Express the area of Dexter's bedroom in square feet and in square yards. 2. Which store offers the best buy on the carpeting? Explain your thinking. Answer: 1. The bedroom has an area of 150 square feet or approximately 16.7 square yards. 2. Store B offers the best buy. The room requires 150 square feet of carpet. Store A sells the carpet for $3.75 per square foot which makes the total cost $562.50. Store B sells the carpet for $25.50 per square yard. Since 16.7 square yards are needed the total cost is $425.85 making Store B the best buy. Or To make it possible to compare the two prices, the price at Store B can be changed to a price per square foot. There are 9 square feet in a square yard so if the $25.50 per square yard price is divided by 9, the cost per square foot is $2.83. So Store B is the best buy.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: A rectangle's length is 60 mm and its perimeter is 160 mm. What is the area of the rectangle in square centimeters? Explain your thinking. Answer: To find the area, the width must be known. Since the perimeter is 160 mm and the length is 60, the width must be 20 mm. The area is found by multiplying 20 by 60 for an area of 1200 square mm. There are 100 square mm in a square cm (10 ● 10) so the area can be expressed as 12 square centimeters. Or The length can be converted to 6 centimeters and the perimeter to 16 cm. The width must be 2 cm. The area can be found by multiplying 6 cm by 2 cm for an area of 12 square centimeters. Problem: A box has dimensions of 15 cm x 10 cm x 8 cm. The volume of the box could be expressed as 1200 cubic centimeters. Express the volume in two other ways using different units. Explain your thinking. Sample answer: The volume could be expressed in cubic millimeters. The dimensions could be written as 150 mm x 100 mm x 80 mm. This means the volume would be 1,200,000 cubic millimeters. Or

CONCEPT	EXPECTATION	EXAMPLE
		The volume of 1200 cubic centimeters could be converted to cubic millimeters. There are 1000 cubic millimeters in each cubic centimeter. 1200 ● 1000 = 1,200,000 cubic millimeters. Or The volume could be expressed in cubic meters. The dimensions could be written as 0.15 m x 0.1 m x 0.08 m. This means the volume would be 0.0012 cubic meters. Or The volume of 1200 cubic centimeters could be converted to cubic meters. There are 1,000,000 cubic centimeters in a cubic meter. 1200 ÷1,000,000 = 0.0012 cubic meters. Or Since one cubic centimeter is equivalent to one milliliter, the volume could be expressed as 1200 milliliters.

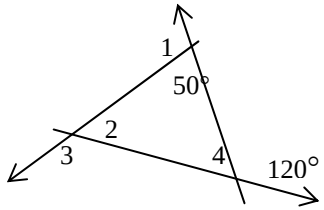
MEASUREMENT

Grade 9

BIG IDEA (1): Understand measurable objects and the units, systems and processes of measurement.

	CONCEPT	EXPECTATION	EXAMPLE
A	Determine unit of measurement	Identify and justify the unit of measure for velocity	Problem: Mr. Al Jabr is issued a speeding ticket for going 60 mph in a 50 mph hour zone. He argues that he is only going 88 ft/sec. If speeding tickets are issued for anyone going 10 mph or more over the given speed limit, should Mr. Al Jabr have been issued a speeding ticket? Justify your answer with the appropriate units of measure. Answer and notes: $88\frac{ft}{\sec} * 60\frac{\sec}{\min} * 60\frac{\min}{hr} * 1\frac{mile}{5280\,ft} = 60mph$ Yes he should be issued the speeding ticket

Big Idea 2: Apply appropriate techniques, tools and formulas to determine measurements

CONCEPT	EXPECTATION	EXAMPLE
B Use angle measures	Solve problems of angle measure, including those involving triangles and other polygones	Problem: Find the measure of ∠1, ∠2, ∠3, and ∠4 from the triangle given below. Show the work that justifies your answers.  <u>ANSWER AND NOTES:</u> ∠4 + 120°= 180°, therefore ∠4 = 60°, ∠1 + 50° = 180°, therefore ∠1 = 130°, ∠4 + ∠2 + 50° = 180°, since ∠4 = 60°, 60° + 50° + ∠2 = 180°, hence ∠2 = 70°. If ∠2 = 70°, then ∠3 must equal 110°.

CONCEPT	EXPECTATION	EXAMPLE
C Apply geometric measurements	Determine the surface area and volume of geometric figures, including cones, spheres and cylinders	Problem: You have two boxes with lids. Which box required more cardboard, a box 8 in by 6.25 in by 10.5 in., or a box 9 in by 5.5 in. by 11.75 in.? Show the work that justifies your solution. Answer: Box 1 (8x6.25 x 10.5) has the following surface area: 8 x 6.25 = 50 8 x 10.5 = 84 6.25 x 10.5= 65.625 199.625 x 2 = 399.25 sq in Box 2 (9 x 5.5 x 11.75) 9 x 5.5 = 49.5 9 x 11.75 = 105.75 5.5x 11.75 = 64.625 219.875 x2 = 439.75 sq in Box 2 would require more cardboard. Problem: Joni has to determine how much it will cost to make a cylinder that is 10 in high and has a diameter of 6 in. The material for the top and bottom costs 1¢

CONCEPT	EXPECTATION	EXAMPLE
		per in^2. The material for the side costs 2¢ in^2. Find the total cost of the material rounded to the nearest cent. (Assume that there will be no waste material). <u>ANSWER AND NOTES:</u> Area of the top and bottom: $2 \bullet \pi (9) = 18\pi$ Area of the side: $2 \bullet \pi \bullet 3 \bullet 10 = 60\pi$ Cost of top and bottom: $1¢ \bullet 18\pi$ Cost of the side: $2¢ \bullet 60\pi$ Total cost = $(2¢ \bullet 60\pi) + (1¢ \bullet 18\pi) = \$4.34$ One possible extension would be to have students think about the following problem: Joni is considering this problem and wonders if she makes a right circular cone, how would the cost compare? She realizes that the cone must hold at least as much as the cylinder. Would this shape cost less? Problem: How much frozen yogurt can you pack inside a cone that is 5 in. high with a radius of 1.25 in.? Answer: This is a volume problem. The volume of a cone is found by multiplying the base area times the height and then dividing by 3.

CONCEPT	EXPECTATION	EXAMPLE
		V = (1.25)(1.25)(3.14)(5)/3 = 24.53125 /3 = 8.2 cu inches of yogurt

	CONCEPT	EXPECTATION	EXAMPLE
D	Analyze precision	Analyze effects of computation on <u>precision</u>	Problem: John wants to buy new carpeting for his bedroom. He measured his room using a meter stick but thought he was using a yardstick. According to his measurements, the room is 4 yards by 5 yards. He goes to the store and finds out that the carpet, pad and installation costs $4 per square foot. He approximates the cost to be $720. Much to his surprise, after the salesman from the store measures, the estimate is $861. Why is John's estimate so far off? Justify your answer. <u>ANSWER AND NOTES:</u> Justifications will vary. Students need to be able to convert square yards to square feet and also realize that a meter stick is more than 3 inches longer than a yardstick. There have been several examples in recent years when two groups work on the same project—one-group measures in metric and the other group use American standard measure. Both groups assuming the other group is using their system of measurement. To further emphasize the importance of precision, the lengths of the sides could be increased, that would show a larger difference in the price. Problem:

CONCEPT	EXPECTATION	EXAMPLE
		In Geometry class, the students were asked to find the area of a circle with a radius of 2.143 inches. They were asked to round their answers to the nearest hundredth. When finished three students had the following results: Martha A = 3.14 x 2.14 x 2.14 = 14.38 sq inches Joan A = π x 2.143 x 2.143 (Joan used her calculator button for pi) = 14.43 sq inches Taniel A = 3.14 x 2.143 x 2.143 = 14.42 sq inches Explain the difference in answers that each student was able to get even though they used the same formula. Answer: Martha began her problem by rounding everything to the nearest hundredth thus leading to a smaller answer than the rest of the students. Joan chose to use here calculator value of pi which is several decimal places long. As a result of not rounding until the end of the problem, Joan was able to get as similar but larger answer to her problem. Taniel, on the other hand, chose to round the value of pi to the nearest hundredth at the beginning but left the radius in thousands as she did the problem. This resulted in an answer that is larger than Martha but less than Joan. This problem shows the effect of rounding at different parts of a problem, The rounding process will change answers even though in the end they will be very close to each other.

CONCEPT	EXPECTATION	EXAMPLE
E Use relationships within a measurement system	Use unit analysis to solve problems involving rates	Problem: You are visiting Mexico and you want to exchange $250 for pesos. The rate of exchange is 9.990 pesos per United states dollar. You want to find out how many pesos you will receive. Answer: You can use unit analysis to write an equation that will help you find the amount of money. $\text{Dollars} \cdot \frac{\text{pesos}}{\text{Dollars}} = \text{pesos}$ $250 \cdot \frac{9.990}{1} = \text{pesos}$ $2497.5 = \text{pesos}$ You will receive 2497 pesos. Problem: How many inches are there in 400 yards? Answer: $\frac{400 \text{ ~~yds~~}}{1} \cdot \frac{3\text{~~ft~~}}{1 \text{ ~~yd~~}} \cdot \frac{12 \text{ inches}}{1 \text{ ~~ft~~}} = \#$ of inches $400 \times 3 \times 12 = \#$ of inches $14400 = \#$ of inches

CONCEPT	EXPECTATION	EXAMPLE
		There are 14400 inches in 400 yards Problem: Megan can read an average of **x** words per minute. If there are an average **y** words per page, how many pages will Megan read in 1 hour? ANSWER AND NOTES: $\frac{\text{x words}}{\text{1 minute}} \times \frac{\text{1 page}}{\text{y words}} \times \frac{\text{60 minutes}}{\text{hour}} = \frac{\text{60} \bullet \text{x pages}}{\text{y hours}}$

precision: an indication of how finely a measurement was made. When you calculate with measured values, you may need to round to the smallest place in the roughest actual measurements. [2]

unit analysis: keeping track of units during computation to assure accurate and appropriate reporting of information.[3]

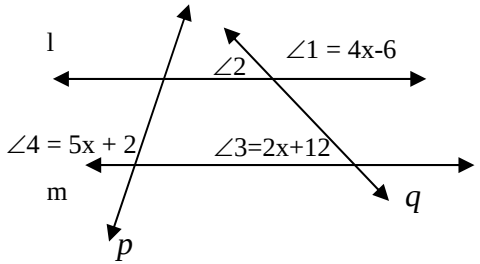
[2] *Algebra to Go: A mathematics handbook* (p. 499). (2000). Wilmington, MA: Great Source Education Group, Inc

[3] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 322). Reston, VA

MEASUREMENT

Grade 10

BIG IDEA (2): Apply appropriate techniques, tools and formulas to determine measurements

	CONCEPT	EXPECTATION	EXAMPLE
B	Use angle measurement	Solve problems of angle measure of parallel lines cut by a transversal	Problem: In the figure below, $l \parallel m$. l and m are cut by transversal p and q. Find the measures of $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. Show your work to justify your angle measurements.  Answer: $\angle 2 \cong \angle 3$ $\angle 1 + \angle 2 = 180$ $4x - 6 + 2x + 12 = 180$ $6x + 6 = 180$ $6x = 174$ $x = 29$ $\angle 1 = 4(29) - 6 = 110^o$ $\angle 2 = 70^o$ $\angle 3 = 2(29) + 12 = 70^o$ $\angle 4 = 5(29) + 2 = 147^o$

CONCEPT	EXPECTATION	EXAMPLE
C Apply geometric measurements	Determine the surface area, and volume of geometric figures, including cones, spheres and cylinders	Problem: 1. Find the surface area of a cone whose radius is 5 cm, height is 12 cm and slant height is 13 cm. Round your answer to the nearest tenth. 2. Find the volume of a cone whose radius is 5 cm and height is 12 cm. Round your answer to the nearest tenth. Answer: 1. $SA = \pi r^2 + \pi rl = \pi(5)^2 + \pi(5)13 = 90\pi = 282.6cm^2$ 2. $V = 1/3\,\pi\, r^2\,\mathrm{h} = 1/3\,\pi\,(5)^2\ 12 = 986.0\ cm^3$ Problem: 1. Find the surface area of a cylinder whose radius is 5 cm and height is 12 cm. Round your answer to the nearest tenth. 2. Find the volume of a cylinder whose radius is 5 cm and height is 12 cm. Round your answer to the nearest tenth. Answer: 1. $SA = 2\pi r^2 + 2\pi rh = 2\pi(5)^2 + 2\pi(5)12 = 170\pi = 533.8cm^2$ 2. $V = \pi\, r^2\,\mathrm{h} = \pi\,(5)^2\ 12 = 942\ cm^3$

Problem:

1. Find the surface area of the largest sphere that will fit inside a cylinder that has a radius of 10cm and a height of 12 cm. Round your answer to the nearest hundredth.

2. Find the volume of the largest sphere that will fit inside a cylinder that has a radius of 10cm and a height of 12 cm. Round your answer to the nearest hundredth.

Answer:

The radius of the sphere inscribed within this cylinder would be 10 cm. Applying the volume and surface area formulas for sphere:

1. $SA = 4\pi r^2 = 4\pi(10)^2 = 400\pi = 1256 cm^2$

2. $V = 4/3\,\pi\, r^3 = 4/3\,\pi\,(10)^3 = (4000/3)\,\pi = 4186.67\ cm^3$

	CONCEPT	EXPECTATION	EXAMPLE
D	Analyze precision	Analyze effects of computations on precision	Problem: Lewis measured the radius of a cylinder to be 4.9 cm and the height to be 12.2 cm. To simplify the problem, Lewis rounded the radius to 5 cm and the height to 12 cm. Compare the measured volume of the cylinder to the estimated volume. Answer: $V = \pi r^2 h$ $\text{measured}: \pi(4.9^2)(12.2) = \pi(292.922) \approx 920.24 \text{ cm}^3$ $\text{estimated}: \pi(5^2)(12) = \pi(300) \approx 942.48 cm^3$ The measures volume is 22.24 cubic centimeters less than the estimated volume. This is calculation difference is a result of the estimated value being rounded up .1 cm. TEACHER NOTES: When precision is important, students should not round their solution(s) until the final step.

DEFINITIONS:

precision: an indication of how finely a measurement was made. When you calculate with measured values, you may need to round to the smallest place in the roughest actual measurements. [1]

[1] *Algebra to Go: A mathematics handbook* (p. 499). (2000). Wilmington, MA: Great Source Education Group, Inc