

# ALGEBRAIC RELATIONSHIPS

## Kindergarten

BIG IDEA (1): Understand patterns, relations and functions

CONCEPT	EXPECTATION	EXAMPLE
<p>A Recognize and extend patterns</p>	<p>Recognize or repeat sequences of sounds or shapes</p>	<p>The teacher will demonstrate a pattern (e.g., triangle, triangle, square) of shapes using pattern blocks. The students will then continue the pattern independently using pattern blocks. The teacher will demonstrate a sound pattern (e.g., clap, clap, stomp). Students will then join the teacher, following the same sound pattern.</p> <p>TEACHER NOTES: Other ways for students to recognize and extend patterns are through repetitive songs, rhythmic chants, and predictive poems that are based on repeating and growing patterns.</p>
<p>B Create and analyze patterns</p>	<p>Create and continue patterns</p>	<p>Students will use pattern blocks to create their own patterns, which must be repeated three times to be evident as a pattern.</p> <p>TEACHER NOTES: Each month, different shapes to represent holidays or special days could be used for students to form patterns. Examples: August—school house, book; September—colored leaves; October—bat, spider, owl; November—turkey, pumpkin, pilgrim; December—evergreen, star, deer; January—snowflake, snowman, bell; February—heart, Lincoln, Washington; March—shamrock (turned in various positions); April—rabbit, egg, raindrop; May—dog, cat, flower. Depending on the ability level of each student, you may want to add or delete shapes. You may want to create a monthly book of patterns for each child to bind and take home at the end of the school year.</p>

CONCEPT	EXPECTATION	EXAMPLE
C Classify objects and representations	Sort objects by size	<p>Students will sort by size objects found in the classroom. Examples: chairs, pencils, various balls (ping-pong, tennis, softball, basketball), shoes, paperclips, etc.</p> <p>TEACHER NOTES:  <i>Homework</i>—Have parents help students find something at home that is sorted by size (e.g., socks), and ask the children to share it with the other students. Make a chart or graph to show the different items found by each family.</p>

BIG IDEA (3): Use mathematical models to represent and understand quantitative relationships

CONCEPT	EXPECTATION	EXAMPLE
<p>A Use mathematical models</p>	<p>Model situations that involve whole numbers, using pictures, objects or symbols</p>	<p>“I want to know if we have enough snacks for everyone who is here today. We have 18 cookies. Will that be enough for everyone to get one cookie? When you have figured out if we have enough or not, be sure you can show me how you know.”</p> <p>Give students pictures of dogs in which to draw the number of spots you specify. For example, “Draw five spots on the first dog. Draw eight spots on the second dog.”</p> <p>TEACHER NOTES: “Students should learn to make models to represent and solve problems.</p> <p>There are six chairs and stools. The chairs have four legs and the stools have three legs. Altogether there are twenty legs. How many chairs and stools [each] are there?</p> <p>One student may represent the situation by drawing six circles and then putting tallies inside [the circles] to represent the number of legs. Another student may represent the situation by using symbols, making a first guess that the number of stools and chairs is the same and adding <math>3 + 3 + 3 + 4 + 4 + 4</math>. Realizing that the sum is too large, the student might adjust the number of chairs and stools so that the sum of their legs is 20.”<sup>1</sup></p>

DEFINITION:

Model—to represent a mathematical situation with manipulatives (objects), pictures, numbers, or symbols.<sup>2</sup>

<sup>1</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 95). Reston, VA: Author.

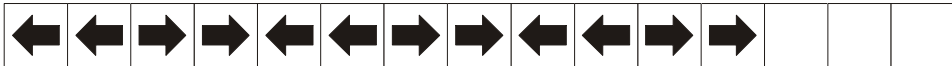

<sup>2</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 95). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
		<p>The example in the Teacher Notes on the previous page is provided to clarify the term “model”; problems for kindergarten students would use smaller numbers, such as the following:</p> <p>There are two dogs and one bird. The dogs have four legs, and the birds have two legs. How many legs are there altogether?</p> <p>Several students will draw pictures and tallies to solve the problem, while some may actually add the numbers.</p>

# ALGEBRAIC RELATIONSHIPS

## Grade 1

BIG IDEA (1): Understand patterns, relations and functions

CONCEPT	EXPECTATION	EXAMPLE
<p>A Recognize and extend patterns</p>	<p>Extend patterns of sound, shape, motion or a simple numeric pattern</p>	<p>The teacher will demonstrate a pattern (e.g., triangle, triangle, square, rhombus) of shapes using pattern blocks. The students will continue the pattern independently using pattern blocks. The teacher will demonstrate a sound pattern (e.g., clap, clap, stomp). Students will then join the teacher, following the same sound pattern. Introduce skip counting using even and odd numbers. Students should complete the next three numbers for each numeric pattern. For example, if the teacher says, “2, 4, 6,...,” students will complete by saying, “8, 10, 12.”</p> <p>Problem:</p>  <p>Draw the arrows in the last three boxes.</p> <p>Answer:</p> 

CONCEPT	EXPECTATION	EXAMPLE
		<p><b>TEACHER NOTES:</b> Although skip counting is a great way for students to identify and extend patterns, there are other ways, too, in which they can recognize and extend patterns, e.g., through repetitive songs, rhythmic chants, and predictive poems that are based on repeating (123, 123, 123,...) or growing patterns (1, 4, 7, 10,...).</p>
<p>B Create and analyze patterns</p>	<p>Describe how simple repeating patterns are generated</p>	<p>Students will communicate what happens in a repeating pattern. For example, in the repeating pattern of triangle, square, circle, the student will communicate that since the triangle was the first shape in the pattern, it will always follow the circle. The square will always follow the triangle, and the circle will always follow the square. Students should transfer this reasoning to all patterns (numbers, words, colors, designs, etc.).</p> <p>Problem: Describe the pattern below, and tell what color the next box will be.</p> <div data-bbox="835 873 1593 938" style="text-align: center;"> </div> <p>Answer: The pattern is white, shaded, shaded. The next box will be white.</p>

**DEFINITION:**

repeating patterns—patterns that are cyclical in nature, with each cycle repeating elements in the same order.

Example: ABCABCABC.<sup>1</sup>

<sup>1</sup> National Council of Teachers of Mathematics. (2001). *Navigating through geometry in prekindergarten–grade 2* (p. 7). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
		<p>Problem: Write the next two numbers in the pattern.</p> <p style="text-align: center;">1, 2, 4, 1, 2, 4, 1, 2, 4, 1, 2, ____, ____</p> <p>Answer: 4, 1</p>
<p>C Classify objects and representations</p>	<p>Classify objects by size or number</p>	<p>Students can classify items in the room by size. They can also classify items by number, using objects that come in twos, threes, fours, etc.</p> <p>TEACHER NOTES: Make students aware of how they might be classified, e.g., by age, shoe size, etc.</p>

BIG IDEA (2): Represent and analyze mathematical situations and structures using algebraic symbols

CONCEPT	EXPECTATION	EXAMPLE
<p>A Represent mathematical situations</p>	<p>Represent a mathematical situation as an expression or number sentence</p>	<p>Problem: There were nine children playing on the swings. Then eight more children came to play on the swings. How many children were playing on the swings? Write a number sentence to help you solve the problem.</p> <p>Answer: <math>9 + 8 = 17</math></p> <p>TEACHER NOTES: Throughout first grade, students should be encouraged to begin to use mathematical symbols to represent mathematical situations.</p> <p>“Through classroom discussions of different representations during the pre-K–2 years, students should develop an increased ability to use the symbols as a means of recording their thinking. In the earliest years, teachers may provide scaffolding for students by modeling for them until they have the ability to record their ideas. Original representations remain important throughout the students’ mathematical study and should be encouraged. Symbolic representations and manipulation should be embedded in instructional experiences as another vehicle for understanding and making sense of mathematics.</p>

DEFINITION:

expression—a mathematical phrase that represents a number through the combination of operation symbols, numbers, and/or symbols. Examples:  $2 + 3$ ;  $5 - 4$ .<sup>2</sup>

<sup>2</sup> *Math at hand: A mathematics handbook* (p. 523). (1999). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		<p>Equality is an important algebraic concept that students must encounter and begin to understand in the lower grades. A common explanation of the equals sign given by students is that ‘the answer is coming,’ but they need to recognize that the equals sign indicates a relationship—that quantities on each side are equivalent, for example, <math>10 = 4 + 6</math> or <math>4 + 6 = 5 + 5</math>.<sup>3</sup></p>

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<sup>3</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 94). Reston, VA: Author.

BIG IDEA (3): Use mathematical models to represent and understand quantitative relationships

CONCEPT	EXPECTATION	EXAMPLE
<p>A Use mathematical models</p>	<p>Model situations that involve the addition of whole numbers, using pictures, objects or symbols</p>	<p>Problem: John has one dog and one duck. How many legs do the two animals have together? Show how you know your answer is correct.</p> <p>Answer:</p> <div data-bbox="856 565 1192 755" style="text-align: center;"> </div> <p><math>4 + 2 = 6</math></p> <p>TEACHER NOTES: “Students should learn to make models to represent and solve problems.</p> <p>There are six chairs and stools. The chairs have four legs and the stools have three legs. Altogether there are twenty legs. How many chairs and stools [each] are there?</p>

DEFINITION:

Model—to represent a mathematical situation with manipulatives (objects), pictures, numbers, or symbols.<sup>4</sup>

<sup>4</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 95). Reston, VA: Author.

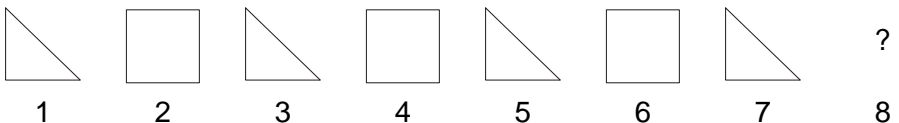
CONCEPT	EXPECTATION	EXAMPLE
		<p>One student may represent the situation by drawing six circles and then putting tallies inside [the circles] to represent the number of legs. Another student may represent the situation by using symbols, making a first guess that the number of stools and chairs is the same and adding <math>3 + 3 + 3 + 4 + 4 + 4</math>. Realizing that the sum is too large, the student might adjust the number of chairs and stools so that the sum of their legs is 20.”<sup>5</sup> <i>Note: This is a good example of a problem that has more than one solution.</i></p> <p>The example above is provided to clarify the term “model.” First-grade problems would include smaller numbers or only portions of the problem as follows:</p> <p>There are six chairs. How many legs are there altogether?</p> <p>Some students will draw all six chairs and count the legs, while others will make six sets of tally marks. Still others will do mental math, visualizing the chairs and counting.</p>

<sup>5</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 95). Reston, VA: Author.


# ALGEBRAIC RELATIONSHIPS

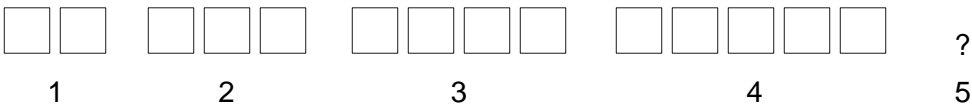
## Grade 2

BIG IDEA (1): Understand patterns, relations and functions

CONCEPT	EXPECTATION	EXAMPLE
A Recognize and extend patterns	Describe and extend simple numeric patterns and change from one representation to another	<p>Teachers should help students develop the ability to form generalizations by asking students questions such as “How could you describe this pattern?”, “How can it be repeated or extended?” or “How are these patterns alike?” For example, students should recognize that the color pattern “blue, blue, red, blue, blue, red” is the same in form as “clap, clap, step, clap, clap, step.” This recognition lays the foundation for the idea that two very different situations can have the same mathematical features and are the same in some important ways. Knowing that each pattern could be described as having the form AABAAB is an early introduction to the power of algebra for students.<sup>1</sup> Teachers can explore functions with students by pairing counting numbers with a repeating pattern, as in the figure below.</p> <div style="text-align: center;">  <p style="margin: 0;">1      2      3      4      5      6      7      8</p> </div>

<sup>1</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 91–92). Reston, VA: Author.
















CONCEPT	EXPECTATION	EXAMPLE
		<p>Problem: Look at the three patterns below and tell how they are alike and different.</p> <p>1. </p> <p>2. A B B A B B A B B A B B</p> <p>3. Clap Snap Snap Clap Snap Snap Clap Snap Snap Clap Snap Snap</p> <p>Answer: They are alike because they are all ABBABBABBABB forms. They are different because one is made of shapes, one is made of letters, and one is made of sounds.</p> <p>TEACHER NOTES: Help students develop effective questioning strategies to form connections about patterns. How could this pattern be extended? How is this pattern similar to other patterns? Help students make generalizations about various patterns. Example: Red, red, green in a paper chain has the same repeating pattern as clap, clap, slap in a listening pattern. Both can be identified as an AAB pattern. Transfer this knowledge of patterns to poetry.</p>

CONCEPT	EXPECTATION	EXAMPLE
B Create and analyze patterns	Describe how simple growing patterns are generated	<p>Problem: How many squares would be in position 5?</p>  <p>Answer: Position 5 would have six squares. The number of squares is always one more than the position.</p> <p>Problem: What is the next number in the sequence?</p> <p>3, 6, 9, _____</p> <p>Answer: 12</p>

**DEFINITIONS:**

growing patterns—patterns that show an arithmetic change between pairs of elements in the pattern. For example, growing patterns may show numbers in decreasing order or buildings in decreasing size. Example: 3, 5, 8, 12,...<sup>2</sup>

<sup>2</sup> Greenes, C., Cavanagh, M., Dacey, L., Findell, C., Small, M. (2001). *Navigating through algebra in prekindergarten–grade 2* (p. 4). Reston, VA: National Council of Teachers of Mathematics.

CONCEPT	EXPECTATION	EXAMPLE										
		<p>Problem:</p> <p>1. Insert the number of feet the ducks in each row have.</p> <table border="1" data-bbox="856 383 1812 1101"> <tbody> <tr> <td data-bbox="856 383 1241 527">1 duck = <u>2</u> feet</td> <td data-bbox="1241 383 1812 527"></td> </tr> <tr> <td data-bbox="856 527 1241 672">2 ducks = _ feet</td> <td data-bbox="1241 527 1812 672"></td> </tr> <tr> <td data-bbox="856 672 1241 816">3 ducks = _ feet</td> <td data-bbox="1241 672 1812 816"></td> </tr> <tr> <td data-bbox="856 816 1241 961">4 ducks = _ feet</td> <td data-bbox="1241 816 1812 961"></td> </tr> <tr> <td data-bbox="856 961 1241 1101">5 ducks = _ feet</td> <td data-bbox="1241 961 1812 1101"></td> </tr> </tbody> </table> <p>2. How many ducks are there in all? How many feet do they have in all?</p> <p>___ ducks = ___ feet</p>	1 duck = <u>2</u> feet		2 ducks = _ feet		3 ducks = _ feet		4 ducks = _ feet		5 ducks = _ feet	
1 duck = <u>2</u> feet												
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4 ducks = _ feet												
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CONCEPT	EXPECTATION	EXAMPLE																								
		<p>Answer:</p> <ol style="list-style-type: none"> <li>2 ducks = 4 feet; 3 ducks = 6 feet; 4 ducks = 8 feet; 5 ducks = 10 feet</li> <li>Total: 15 ducks = 30 feet</li> </ol> <p>Problem:</p> <p>Find the pattern in the following T-chart to fill in the missing numbers. Write about the pattern.</p> <table border="1" data-bbox="856 646 1430 930"> <thead> <tr> <th>Number of ducks</th> <th>Number of feet</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> </tr> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>3</td> <td></td> </tr> <tr> <td>4</td> <td></td> </tr> <tr> <td>5</td> <td></td> </tr> </tbody> </table> <p>Answer:</p> <table border="1" data-bbox="856 1044 1430 1328"> <thead> <tr> <th>Number of ducks</th> <th>Number of feet</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> </tr> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>3</td> <td>6</td> </tr> <tr> <td>4</td> <td>8</td> </tr> <tr> <td>5</td> <td>10</td> </tr> </tbody> </table> <p>The pattern is add two more feet than the duck before.</p>	Number of ducks	Number of feet	1	2	2	4	3		4		5		Number of ducks	Number of feet	1	2	2	4	3	6	4	8	5	10
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CONCEPT	EXPECTATION	EXAMPLE
C Classify objects and representations	Classify objects by size, number or other attributes	Use attribute blocks and the following activity to focus student attention on attributes to solve a problem. Choose a block without students knowing which block it is. Give each student, pair of students, or other cooperative group a different block from the one you've chosen. Have them describe their block by its attributes (e.g., thin, blue, small circle). Tell students how many attributes it has in common with the secret shape you've chosen. If your block is a small, thick, red triangle, you would tell the students that their block has one attribute in common with your secret shape. After three groups of students have shown their shapes and you've told them the number of attributes each of those shapes has in common with your secret shape, allow students to guess the secret shape.

**DEFINITIONS:**

attributes—a characteristic or distinctive feature—such as shape, size, color—of an object or given set of objects.<sup>3</sup>

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<sup>3</sup> Eather, J. *A math dictionary for kids*. Retrieved June 5, 2004, from [www.amathsdictionaryforkids.com](http://www.amathsdictionaryforkids.com).

CONCEPT	EXPECTATION	EXAMPLE
		<p data-bbox="787 276 1785 349">Problem: Place each of the following shapes in the correct place in the diagram below.</p> <div data-bbox="861 381 1722 527"> </div> <div data-bbox="882 617 1743 1104"> <p data-bbox="1092 617 1228 657">Triangles</p> <p data-bbox="1417 617 1512 657">White</p> </div> <p data-bbox="787 1209 1869 1315">Answer: White triangle in the middle, white hexagon in the white region, and shaded triangle in the triangle region.</p>

BIG IDEA (2): Represent and analyze mathematical situations and structures using algebraic symbols

CONCEPT	EXPECTATION	EXAMPLE
A Represent mathematical situations	Represent a mathematical situation as an expression or number sentence	<p>Problem: Sue had 16 pieces of candy. After she gave Ellen some candy, she had 12 pieces left. Use numbers and symbols to write a problem about Sue’s candy.</p> <p>Answer: <math>12 = 16 - \square</math> or <math>16 - \square = 12</math></p> <p>Problem: Bob had 12 goldfish. For his birthday, he got 5 more. How many goldfish does he have now?</p> <p>Answer: <math>12 + 5 = 17</math></p>

DEFINITIONS:

expression—a mathematical phrase that represents a number through the combination of operation symbols, numbers, and/or symbols. Examples:  $2 + 5$ ;  $4 - 2$ .<sup>4</sup>

<sup>4</sup> *Math at hand: A mathematics handbook* (p. 523). (1999). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		<p>Problem: Sam was playing marbles with his friends. He started with 16 marbles. At the end of the game, he had 48 marbles. How many marbles did Sam win? Write a number sentence to help you solve the problem.</p> <p>Answer: <math>48 - 16 = 32</math></p> <p>Problem: Bo and his friends went on rides at the amusement park. There were 14 riders on the first ride, 18 riders on the second ride, 22 riders on the third ride, and 26 riders on the fourth ride. If this pattern continues, how many riders will be on the 10<sup>th</sup> ride?</p> <p>Answer: 50 riders</p> <p>TEACHER NOTES: Throughout second grade, students should be encouraged to begin to use mathematical symbols to represent mathematical situations.</p> <p>“Through classroom discussions of different representations during the pre-K–2 years, students should develop an increased ability to use the symbols as a means of recording their thinking. In the earliest years, teachers may provide scaffolding for students by writing for them until they have the ability to record their ideas. Original representations remain important throughout the students’ mathematical study and should be encouraged. Symbolic representations and manipulation should be embedded in instructional experiences as another vehicle for understanding and making sense of mathematics.</p>

CONCEPT	EXPECTATION	EXAMPLE
		<p>Equality is an important algebraic concept that students must encounter and begin to understand in the lower grades. A common explanation of the equals sign given by students is that ‘the answer is coming,’ but they need to recognize that the equals sign indicates a relationship—that quantities on each side are equivalent, for example, <math>10 = 4 + 6</math> or <math>4 + 6 = 5 + 5</math>.<sup>5</sup></p>

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<sup>5</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 159–160). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
B Describe and use mathematical manipulation	Investigate commutative principle with whole numbers	<p>Roll a die several times, recording the numbers rolled. Ask students to find the sum of the numbers rolled. Ask if the sum will be different if the numbers are added in a different order.</p> <p>Give students problems such as the following:</p> $\begin{array}{cccc} 2 + 3 = & 3 + 2 = & 23 + 24 = & 24 + 23 = \\ 18 + 17 = & 17 + 18 = & 5 + 3 = & 3 + 5 = \end{array}$ <p>Ask the students if they notice any similarities in the answers to the problems. Discuss the commutative property of addition.</p>

**DEFINITION:**

commutative principle (law, rule or property)—In addition and multiplication, numbers may be added or multiplied in any order.<sup>6</sup>

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<sup>6</sup> Eather, J. *A math dictionary for kids*. Retrieved August 6, 2004, from [www.amathsdictionaryforkids.com](http://www.amathsdictionaryforkids.com).

CONCEPT	EXPECTATION	EXAMPLE																												
		<p data-bbox="785 277 1860 383">Problem: Fill in the table below using five counters and a handout with two shapes. Place the counters on the shapes to find the missing numbers.</p> <table border="1" data-bbox="854 420 1428 803"> <thead> <tr> <th data-bbox="854 420 1142 516">Number of counters on shape 1</th> <th data-bbox="1142 420 1428 516">Number of counters on shape 2</th> </tr> </thead> <tbody> <tr> <td data-bbox="854 516 1142 565">0</td> <td data-bbox="1142 516 1428 565">5</td> </tr> <tr> <td data-bbox="854 565 1142 613">1</td> <td data-bbox="1142 565 1428 613"></td> </tr> <tr> <td data-bbox="854 613 1142 662"></td> <td data-bbox="1142 613 1428 662">3</td> </tr> <tr> <td data-bbox="854 662 1142 711">3</td> <td data-bbox="1142 662 1428 711">2</td> </tr> <tr> <td data-bbox="854 711 1142 760"></td> <td data-bbox="1142 711 1428 760">1</td> </tr> <tr> <td data-bbox="854 760 1142 803">5</td> <td data-bbox="1142 760 1428 803"></td> </tr> </tbody> </table> <p data-bbox="785 846 890 873">Answer:</p> <table border="1" data-bbox="854 914 1428 1297"> <thead> <tr> <th data-bbox="854 914 1142 1010">Number of counters on shape 1</th> <th data-bbox="1142 914 1428 1010">Number of counters on shape 2</th> </tr> </thead> <tbody> <tr> <td data-bbox="854 1010 1142 1058">0</td> <td data-bbox="1142 1010 1428 1058">5</td> </tr> <tr> <td data-bbox="854 1058 1142 1107">1</td> <td data-bbox="1142 1058 1428 1107">4</td> </tr> <tr> <td data-bbox="854 1107 1142 1156">2</td> <td data-bbox="1142 1107 1428 1156">3</td> </tr> <tr> <td data-bbox="854 1156 1142 1205">3</td> <td data-bbox="1142 1156 1428 1205">2</td> </tr> <tr> <td data-bbox="854 1205 1142 1253">4</td> <td data-bbox="1142 1205 1428 1253">1</td> </tr> <tr> <td data-bbox="854 1253 1142 1297">5</td> <td data-bbox="1142 1253 1428 1297">0</td> </tr> </tbody> </table>	Number of counters on shape 1	Number of counters on shape 2	0	5	1			3	3	2		1	5		Number of counters on shape 1	Number of counters on shape 2	0	5	1	4	2	3	3	2	4	1	5	0
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CONCEPT	EXPECTATION	EXAMPLE
		<p>TEACHER NOTES:            Opportunities to explore the commutative property of addition will often arise when students are allowed to use their own strategies for adding several two-digit numbers. (When they have three or more numbers, they will also use the associative property of addition.)</p> <p>Although it is not necessary to introduce vocabulary such as commutativity and associativity, teachers should be aware of algebraic properties used by students at this age.</p>

DEFINITIONS:

associative property of addition —The sum stays the same when the grouping of three or more addends is changed.

Example:  $(7 + 4) + 3 = 7 + (4 + 3)$ .<sup>7</sup>

commutative property of addition —The sum stays the same when the order of the addends is changed. Example:

$6 + 4 = 4 + 6$ .<sup>8</sup>

<sup>7</sup> *Math at hand: A mathematics handbook* (p. 517). (1999). Wilmington, MA: Great Source Education Group, Inc.

<sup>8</sup> *Math at hand: A mathematics handbook* (p. 519). (1999). Wilmington, MA: Great Source Education Group, Inc.

BIG IDEA (3): Use mathematical models to represent and understand quantitative relationships

CONCEPT	EXPECTATION	EXAMPLE
<p>A Use mathematical models</p>	<p>Model situations that involve addition and subtraction of whole numbers, using pictures, objects or symbols</p>	<p>Teachers could pose problems such as the following:</p> <p>Rex had 12 pieces of candy. During the day, he ate some of the candy. By the end of the day, he had 4 pieces of candy left. How many pieces of candy did he eat that day? Show how you know your answer is correct.</p> <p>There were eight bears eating at the zoo. Then nine more bears came to eat. How many bears were eating at the zoo? Show how you know your answer is correct.</p> <p>TEACHER NOTES:                      “Students should learn to make models to represent and solve problems.”<sup>9</sup></p> <p>Teachers should pose problems such as the following:</p> <p>John is playing with his toys. Each action figure has two legs, and each animal has four legs. Altogether, there are 18 legs. How many action figures and animals does John have?</p>

DEFINITION:

Model—to represent a mathematical situation with manipulatives (objects), pictures, numbers, or symbols.<sup>10</sup>

<sup>9</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 95). Reston, VA: Author.

<sup>10</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 95). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
		<p>Students may represent the situation in different ways. Some may draw a picture representing the number of legs. Others may represent the situation using symbols, making a first guess, then adjusting the number of action figures and animals so that the sum of the legs is 18. The examples following show multiple solutions:  <math>2 + 4 + 4 + 4 + 4</math>; <math>2 + 2 + 2 + 4 + 4 + 4</math>; <math>2 + 2 + 2 + 2 + 2 + 4 + 4</math>; or  <math>2 + 2 + 2 + 2 + 2 + 2 + 4</math>.</p>

BIG IDEA (4): Analyze change in various contexts

CONCEPT	EXPECTATION	EXAMPLE
A Analyze change	Describe qualitative change, such as students growing taller	<p>Problem: Two students weighed their dogs. Bill's dog weighed 23 pounds, and Becky's dog weighed 25 pounds. Without using numbers, write a sentence to compare the weight of Bill and Becky's dogs.</p> <p>Answer: Bill's dog weighs more than Becky's dog; <i>or</i>, Becky's dog weighs less than Bill's dog.</p> <p>During a discussion of data representations, ask students to describe the data without using numbers. This should bring out students' use of comparative words, such as lighter, thinner, etc.</p>

DEFINITION:

qualitative change—a change (in the quality of something) that can be described by words such as taller, shorter, darker, lighter, warmer, etc.<sup>11</sup>

<sup>11</sup> Greenes, C., Cavanagh, M., Dacey, L., Findell, C., Small, M. (2001). *Navigating through algebra in prekindergarten–grade 2* (p. 4). Reston, VA: National Council of Teachers of Mathematics.

CONCEPT	EXPECTATION	EXAMPLE
		<p>TEACHER NOTES:</p> <p>“From a very early age, children recognize examples of change in their environment and describe change in qualitative terms, such as getting taller, colder, darker, or heavier. By measuring and comparing quantities, such as when keeping track of variations in temperature or growth of a classroom plant or pet, children also learn to describe change quantitatively.”<sup>12</sup></p> <p>“In prekindergarten through grade 2, students can, at first, describe qualitative change (‘I grew taller over the summer’) and then quantitative change (‘I grew two inches in the last year’).”<sup>13</sup></p>

<sup>12</sup> Greenes, C., Cavanagh, M., Dacey, L., Findell, C., Small, M. (2001). *Navigating through algebra in prekindergarten–grade 2* (p. 4). Reston, VA: National Council of Teachers of Mathematics.

<sup>13</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 40). Reston, VA: Author

# ALGEBRAIC RELATIONSHIPS

## Grade 3

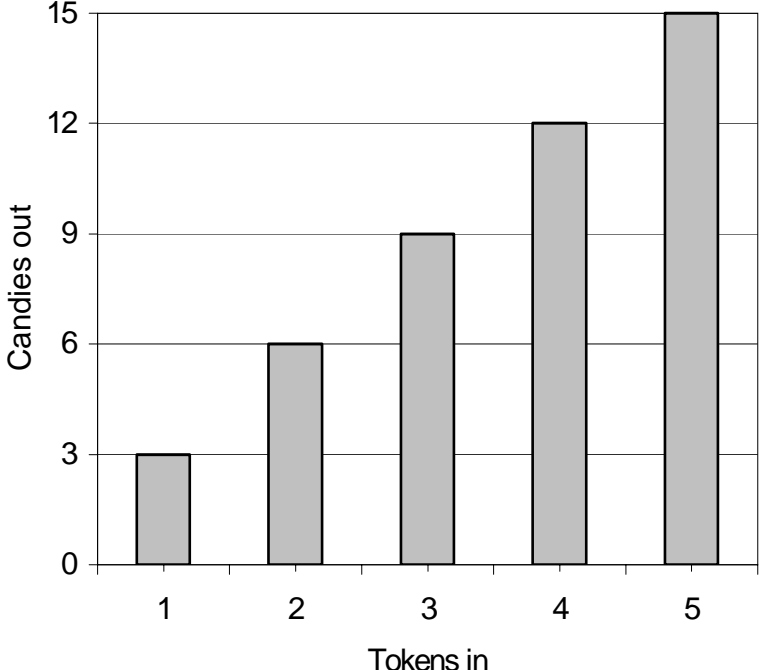
BIG IDEA (1): Understand patterns, relations and functions

CONCEPT	EXPECTATION	EXAMPLE
A Recognize and extend patterns	Extend geometric (shapes) and numeric patterns to find the next term	<p>Problem: Draw the next figure in the pattern below:</p> <p style="text-align: center;">○ □ □ ○ □ □ ○ □ □ ○ □ ____</p> <p>Answer: □</p> <p>Problem: What number would come next in the following pattern?</p> <p style="text-align: center;">2, 2, 3, 3, 4, 4, ____</p> <p>Answer: 5</p> <p>TEACHER NOTES: Although algebra is not a commonly heard word in the 3–5 classroom, the mathematical investigations and conversations of students in these grades</p>

CONCEPT	EXPECTATION	EXAMPLE
		<p>frequently include elements of algebraic reasoning. These experiences not only provide rich contexts for advancing mathematical understanding, they also serve as an important precursor to the more formalized study of algebra in the middle and secondary grades. “Algebraic ideas should emerge and be investigated in grades 3–5 as students</p> <ul style="list-style-type: none"> <li>• identify or build numerical and geometric patterns</li> <li>• describe patterns verbally and represent them with tables or symbols</li> <li>• look for and apply relationships between varying quantities to make predictions</li> <li>• make and explain generalizations that seem to always work in a particular situation</li> <li>• use graphs to describe patterns and make predictions</li> <li>• explore number properties</li> <li>• use invented notation, standard symbols, and variables to express a pattern, generalization, or situation”<sup>1</sup></li> </ul>

<sup>1</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 159–160). Reston, VA: Author

CONCEPT	EXPECTATION	EXAMPLE												
B Create and analyze patterns	Represent patterns using words, tables or graphs	<p>Problem: Lori has discovered a new game at the amusement park. If she puts in 1 token, she receives three candies; if she puts in 2 tokens, she receives six candies; if she puts in 3 tokens, she receives nine candies. Complete the table below to determine how many candies she would receive for each token amount, then make a bar graph to show the information on the table.</p> <table border="1" data-bbox="858 527 1335 816"> <thead> <tr> <th data-bbox="858 527 1094 576">Tokens</th> <th data-bbox="1094 527 1335 576">Candies</th> </tr> </thead> <tbody> <tr> <td data-bbox="858 576 1094 625">1</td> <td data-bbox="1094 576 1335 625">3</td> </tr> <tr> <td data-bbox="858 625 1094 673">2</td> <td data-bbox="1094 625 1335 673">6</td> </tr> <tr> <td data-bbox="858 673 1094 722">3</td> <td data-bbox="1094 673 1335 722">9</td> </tr> <tr> <td data-bbox="858 722 1094 771">4</td> <td data-bbox="1094 722 1335 771"></td> </tr> <tr> <td data-bbox="858 771 1094 816">5</td> <td data-bbox="1094 771 1335 816"></td> </tr> </tbody> </table>	Tokens	Candies	1	3	2	6	3	9	4		5	
Tokens	Candies													
1	3													
2	6													
3	9													
4														
5														

CONCEPT	EXPECTATION	EXAMPLE												
		<p>Answer: For 4 tokens she would get twelve candies, and for 5 tokens, she would get fifteen candies.</p> <p style="text-align: center;"><b>Candy Received Per Token</b></p>  <table border="1" data-bbox="871 527 1627 1193"> <caption>Candy Received Per Token</caption> <thead> <tr> <th>Tokens in</th> <th>Candies out</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>3</td> </tr> <tr> <td>2</td> <td>6</td> </tr> <tr> <td>3</td> <td>9</td> </tr> <tr> <td>4</td> <td>12</td> </tr> <tr> <td>5</td> <td>15</td> </tr> </tbody> </table>	Tokens in	Candies out	1	3	2	6	3	9	4	12	5	15
Tokens in	Candies out													
1	3													
2	6													
3	9													
4	12													
5	15													

CONCEPT	EXPECTATION	EXAMPLE												
		<p data-bbox="787 272 907 300">Problem:</p> <p data-bbox="787 310 1850 375">Find the missing numbers in the table below that shows the number of horses and riders.</p> <table border="1" data-bbox="856 415 1814 513"> <tbody> <tr> <td data-bbox="856 415 1094 464">Horses</td> <td data-bbox="1094 415 1241 464">1</td> <td data-bbox="1241 415 1388 464">2</td> <td data-bbox="1388 415 1535 464">3</td> <td data-bbox="1535 415 1682 464">4</td> <td data-bbox="1682 415 1814 464">5</td> </tr> <tr> <td data-bbox="856 464 1094 513">Riders</td> <td data-bbox="1094 464 1241 513">2</td> <td data-bbox="1241 464 1388 513">4</td> <td data-bbox="1388 464 1535 513">6</td> <td data-bbox="1535 464 1682 513"></td> <td data-bbox="1682 464 1814 513"></td> </tr> </tbody> </table> <p data-bbox="787 553 898 581">Answer:</p> <p data-bbox="787 591 856 618">8, 10</p>	Horses	1	2	3	4	5	Riders	2	4	6		
Horses	1	2	3	4	5									
Riders	2	4	6											

BIG IDEA (2): Represent and analyze mathematical situations and structures using algebraic symbols

CONCEPT	EXPECTATION	EXAMPLE
A Represent mathematical situations	Represent a mathematical situation as an expression or number sentence	<p>Problem: Write a number sentence for “what number added to five equals eight?”.</p> <p>Answer: <i>Answers may vary. Examples: <math>n + 5 = 8</math>; <math>5 + n = 8</math>; <math>\square + 5 = 8</math>; <math>5 + \square = 8</math></i></p> <p>Problem: Write twenty-one plus two as an expression.</p> <p>Answer: <math>21 + 2</math></p>

DEFINITIONS:

expression—a mathematical phrase that represents a number through the combination of operation symbols, numbers, and/or symbols. Examples:  $2 \times 60$ ;  $3 + \Delta$ .<sup>2</sup>

number sentence—equations for comparisons. Examples:  $3 + 4 = 7$ ;  $8 - 2 = 6$ ;  $7 > 6$ .<sup>3</sup>

<sup>2</sup> *Math at hand: A mathematics handbook* (p. 523). (1999). Wilmington, MA: Great Source Education Group, Inc.

<sup>3</sup> Cavanagh, M. (2002). *Math to learn* (p. 457). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		<p>Problem: Use numbers to write twelve minus three equals nine.</p> <p>Answer: <math>12 - 3 = 9</math></p> <p>Problem: Twelve students were playing soccer. At the end of the game, nine players were left. Write a number sentence using the soccer players.</p> <p>Answer: <math>12 - 9 = 3</math></p> <p><b>TEACHER NOTES:</b> Expressions and number sentences can both contain a combination of variables, numbers, and operation symbols. An expression represents a mathematical relationship. A number sentence or equation contains an equal sign indicating that the amount on one side of the equal sign has the same value as the amount on the other side.</p>

CONCEPT	EXPECTATION	EXAMPLE
B Describe and use mathematical manipulation	Apply the commutative property of addition of whole numbers	<p>Problem: Which of the following number sentences gives the same answer as <math>3 + 5</math>?</p> <p>A. <math>3 \times 5</math>            B. <math>5 \times 3</math>            C. <math>5 + 3</math>            D. <math>5 - 3</math></p> <p>Answer: C</p> <p>Problem: Fill in the blank to show the commutative property of addition.</p> <p><math>8 + 4 = \underline{\quad} + 8</math></p> <p>Answer: 4</p>

**DEFINITIONS:**

commutative property of addition—the sum stays the same when the order of the addends is changed.

Example:  $6 + 4 = 4 + 6$ .<sup>4</sup>

<sup>4</sup> *Math at hand: A mathematics handbook* (p. 519). (1999). Wilmington, MA: Great Source Education Group, Inc.

BIG IDEA (3): Use mathematical models to represent and understand quantitative relationships

CONCEPT	EXPECTATION	EXAMPLE
<p>A Use mathematical models</p>	<p>Model problem situations, including multiplication with objects and drawings</p>	<p>Problem: Model the sets below with counters, then write a multiplication sentence for each.</p> <p>1. 5 groups of 2 2. 3 groups of 6</p> <p>Answer: 1. ○ ○ ○ ○ ○ ○ ○ ○ ○ ○                      <math>5 \times 2 = 10</math> 2. ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇                      <math>3 \times 6 = 18</math></p> <p>Problem: Write a multiplication sentence for the following picture:</p> <p>😊😊😊😊😊😊 😊😊😊😊😊😊 😊😊😊😊😊😊</p> <p>Answer: <math>3 \times 6 = 18</math></p>

DEFINITION:

Model—to represent a mathematical situation with manipulatives (objects), pictures, numbers, or symbols.<sup>5</sup>

<sup>5</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 95). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
		<p>TEACHER NOTES:            Students in grades 3–5 can model a variety of situations, including geometric patterns, real-world situations, and scientific experiments. Sometimes they will use their model to predict the next element in a pattern. At other times, they may make a general statement about how one variable is related to another variable: for example, if a sandwich costs \$3, you can figure out how much any number of sandwiches costs by multiplying that number by 3. In modeling situations that involve real-world data, students need to know that their predictions may not always match the observed outcomes for a variety of different reasons. Students should also begin to understand that different models for the same situation can give the same results.<sup>6</sup></p>

<sup>6</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 162–163). Reston, VA: Author.

BIG IDEA (4): Analyze change in various contexts

CONCEPT	EXPECTATION	EXAMPLE
A Analyze change	Describe quantitative change, such as students growing two inches in a year	Problem: Bob's dog weighed 8 pounds when he got him last year. The dog now weighs 25 pounds. Write a sentence that describes how much weight the dog gained since last year.  Answer: Bob's dog has gained 17 pounds since last year.

DEFINITION:

quantitative—relating to number or quantity; elements can be counted or measured.<sup>7</sup>


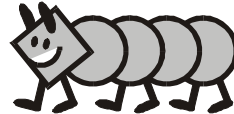
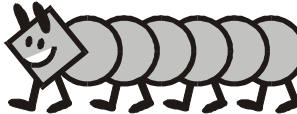

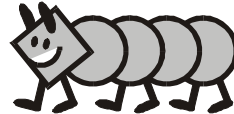
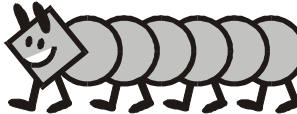

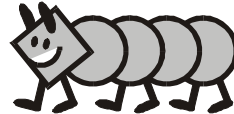
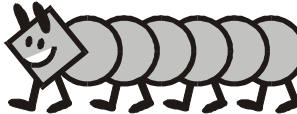
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<sup>7</sup> Eather, J. *A math dictionary for kids*. Retrieved June 5, 2004, from [www.amathsdictionaryforkids.com](http://www.amathsdictionaryforkids.com).

# ALGEBRAIC RELATIONSHIPS

## Grade 4

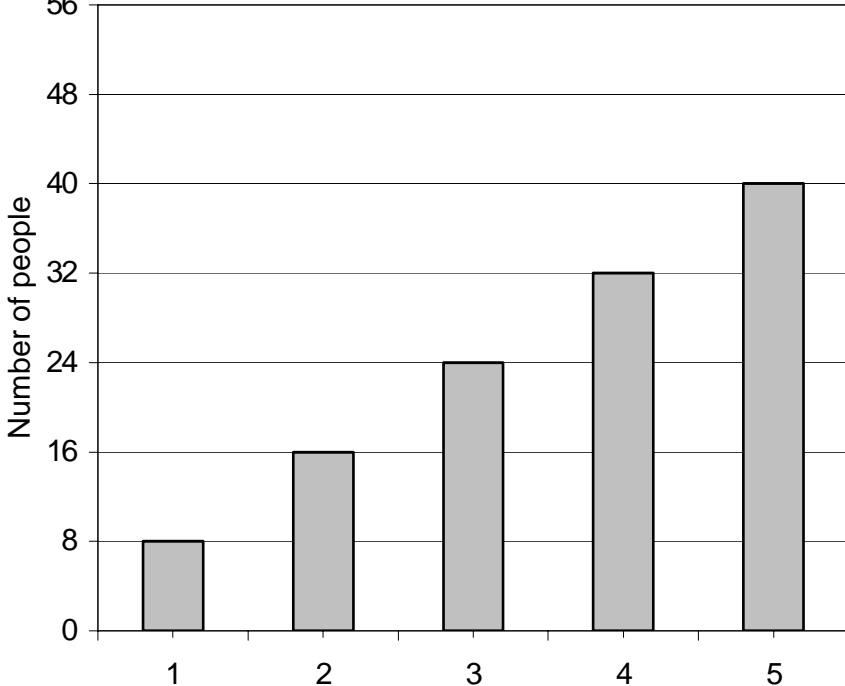
BIG IDEA (1): Understand patterns, relations and functions

CONCEPT	EXPECTATION	EXAMPLE						
A Recognize and extend patterns	Describe geometric and numeric patterns	<p data-bbox="787 527 1816 625">Problem: Based on the information in the graphic below, how many circles would it take to make a 30-day-old bug? Explain how you got your answer.</p> <table border="1" data-bbox="850 665 1795 901"><thead><tr><th data-bbox="850 665 1144 714">one-day-old bug</th><th data-bbox="1144 665 1459 714">two-day-old bug</th><th data-bbox="1459 665 1795 714">three-day-old bug</th></tr></thead><tbody><tr><td data-bbox="850 714 1144 901"></td><td data-bbox="1144 714 1459 901"></td><td data-bbox="1459 714 1795 901"></td></tr></tbody></table> <p data-bbox="787 950 1879 1047">Answer: 60 circles. It takes two circles for each day's age, so for a thirty-day-old bug, it would be <math>30 \times 2 = 60</math> circles.</p> <p data-bbox="787 1136 1648 1201">Problem: Describe the pattern below, and write the number that will go next.</p> <p data-bbox="850 1242 1197 1274">1, 2, 4, 8, 16, 32, 64, _____</p>	one-day-old bug	two-day-old bug	three-day-old bug			
one-day-old bug	two-day-old bug	three-day-old bug						
								

CONCEPT	EXPECTATION	EXAMPLE
		<p>Answer: To get the next number in this pattern, you double or multiply the one before by 2. The next number here will be 128.</p> <p>TEACHER NOTES: Although algebra is not a commonly heard word in the 3–5 classroom, the mathematical investigations and conversations of students in these grades frequently include elements of algebraic reasoning. These experiences not only provide rich contexts for advancing mathematical understanding, they also serve as an important precursor to the more formalized study of algebra in the middle and secondary grades. “Algebraic ideas should emerge and be investigated in grades 3–5 as students</p> <ul style="list-style-type: none"> <li>• identify or build numerical and geometric patterns</li> <li>• describe patterns verbally and represent them with tables or symbols</li> <li>• look for and apply relationships between varying quantities to make predictions</li> <li>• make and explain generalizations that seem to always work in a particular situation</li> <li>• use graphs to describe patterns and make predictions</li> <li>• explore number properties</li> <li>• use invented notation, standard symbols, and variables to express a pattern, generalization, or situation”<sup>1</sup></li> </ul>

<sup>1</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 159–160). Reston, VA: Author.

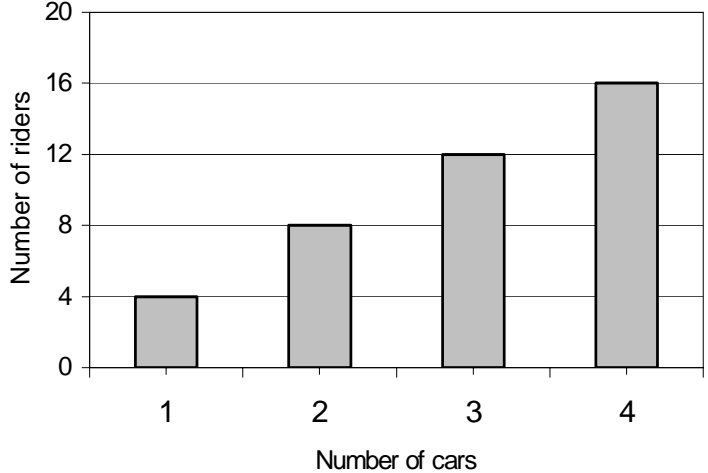
CONCEPT	EXPECTATION	EXAMPLE
B Create and analyze patterns	Analyze patterns using words, tables and graphs	<p>Problem:            At a new after-school program, 4 students attended on Monday, 8 students on Tuesday, and 12 students on Wednesday. If this pattern continues, how many students would attend the after school program on Thursday?</p> <p>Answer:            16 students. You add 4 students to the previous number to continue the pattern.</p>

CONCEPT	EXPECTATION	EXAMPLE												
		<p data-bbox="785 272 1822 378"> <b>Problem:</b>            Using the graph below, determine how many ferry boats would be needed for 48 people. Explain how you got your answer.         </p> <div data-bbox="871 451 1711 1263" style="text-align: center;"> <p data-bbox="1037 451 1512 483"><b>Ferries Needed to Transport People</b></p>  <table border="1" data-bbox="871 527 1711 1209"> <caption>Data from the bar graph</caption> <thead> <tr> <th>Number of ferries</th> <th>Number of people</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>8</td> </tr> <tr> <td>2</td> <td>16</td> </tr> <tr> <td>3</td> <td>24</td> </tr> <tr> <td>4</td> <td>32</td> </tr> <tr> <td>5</td> <td>40</td> </tr> </tbody> </table> </div> <p data-bbox="785 1291 1848 1388"> <b>Answer:</b>            6 ferries would be needed for 48 people. It takes 1 ferry for every 8 people, and 48 divided by 8 equals 6.         </p>	Number of ferries	Number of people	1	8	2	16	3	24	4	32	5	40
Number of ferries	Number of people													
1	8													
2	16													
3	24													
4	32													
5	40													

CONCEPT	EXPECTATION	EXAMPLE
		<p data-bbox="787 272 1491 341">Problem: Explain how the patterns below are alike and different.</p> <div data-bbox="850 389 1522 503"> </div> <div data-bbox="850 544 1438 641"> </div> <div data-bbox="850 690 1417 779"> </div> <p data-bbox="787 876 1879 1023">Answer: They are all repeating patterns with two shapes. In the first two, different shapes are used to make the pattern. In the third one, different colors of the same shape are used to make the pattern.</p>

CONCEPT	EXPECTATION	EXAMPLE																				
		<p>Problem: Complete the table below to show the pattern. Draw a graph to represent the data in the table.</p> <table border="1" data-bbox="856 440 1812 535"> <tr> <td>Cars</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>Number of Riders</td> <td>4</td> <td>8</td> <td></td> <td></td> </tr> </table> <p>Answer:</p> <table border="1" data-bbox="856 634 1812 730"> <tr> <td>Cars</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>Number of Riders</td> <td>4</td> <td>8</td> <td>12</td> <td>16</td> </tr> </table> <p><i>Numerous student solutions are possible for the graphs. Examples:</i></p>	Cars	1	2	3	4	Number of Riders	4	8			Cars	1	2	3	4	Number of Riders	4	8	12	16
Cars	1	2	3	4																		
Number of Riders	4	8																				
Cars	1	2	3	4																		
Number of Riders	4	8	12	16																		

CONCEPT	EXPECTATION	EXAMPLE										
		<p data-bbox="1142 305 1314 329" style="text-align: center;"><b>Riders Per Car</b></p> <table border="1" data-bbox="890 375 1598 902"><caption>Data points from the 'Riders Per Car' graph</caption><thead><tr><th>Number of cars</th><th>Number of riders</th></tr></thead><tbody><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>12</td></tr><tr><td>4</td><td>16</td></tr></tbody></table>	Number of cars	Number of riders	1	4	2	8	3	12	4	16
Number of cars	Number of riders											
1	4											
2	8											
3	12											
4	16											

CONCEPT	EXPECTATION	EXAMPLE										
		<p style="text-align: center;"><b>Riders Per Car</b></p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <caption>Data for Riders Per Car</caption> <thead> <tr> <th>Number of cars</th> <th>Number of riders</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>4</td> </tr> <tr> <td>2</td> <td>8</td> </tr> <tr> <td>3</td> <td>12</td> </tr> <tr> <td>4</td> <td>16</td> </tr> </tbody> </table>	Number of cars	Number of riders	1	4	2	8	3	12	4	16
Number of cars	Number of riders											
1	4											
2	8											
3	12											
4	16											

BIG IDEA (2): Represent and analyze mathematical situations and structures using algebraic symbols

CONCEPT	EXPECTATION	EXAMPLE
A Represent mathematical situations	Represent a mathematical situation as an expression or number sentence	<p>Problem: Five people want to share 20 pieces of candy. Write an expression that shows how to share the candy equally.</p> <p>Answer: <math>20 \div 5</math></p> <p>Problem: Allan has enough dog treats to give each of his dogs three treats. Write an expression to show how many dog treats he has.</p> <p>Answer: <math>\square \times 3</math> or <math>3 \times \square</math> or <math>d \times 3</math> or <math>3 \times d</math> where <math>\square</math> or <math>d</math> represents the number of dogs</p> <p>Problem: Write a number sentence for twelve times three minus two, then find the answer.</p> <p>Answer: <math>12 \times 3 - 2 = 34</math></p>

DEFINITIONS:

expression—a mathematical phrase that represents a number through the combination of operation symbols, numbers, and/or symbols. Examples:  $23 \times 67$ ;  $33 - \Delta$ .<sup>2</sup>

number sentence—an equation or comparison. Examples:  $3 + 4 = 7$ ,  $8 - 2 = 6$ ,  $7 > 6$ .<sup>3</sup>

<sup>2</sup> Cavanagh, M. (2000). *Math to know* (p. 450). Wilmington, MA: Great Source Education Group, Inc.

<sup>3</sup> Cavanagh, M. (2002). *Math to learn* (p. 457). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		<p><b>TEACHER NOTES:</b>            Expressions and number sentences can both contain a combination of variables, numbers, and operation symbols. An expression represents a mathematical relationship. A number sentence or equation contains an equal sign indicating that the amount on one side of the equal sign has the same value as the amount on the other side.</p>

CONCEPT	EXPECTATION	EXAMPLE
B Describe and use mathematical manipulation	Apply the commutative property of multiplication of whole numbers	<p>Problem: Joann and Steve have been discussing how to arrange the 24 chairs for the school play. Joann says they can have three rows of eight. Steve said they can have eight rows of three. Who is correct? Write number sentences or draw a picture to explain your thinking.</p> <p>Answer:</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/>  <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/>  <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/>  <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/>  <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/>  <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/>  <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/>  <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/>  <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/> </div> <div style="text-align: center;"> <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/>  <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/>  <input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/><input type="checkbox"/> </div> </div> <p><math>3 \times 8 = 24</math> and <math>8 \times 3 = 24</math>, so both are correct.</p>

**DEFINITIONS:**

commutative property of multiplication—the product stays the same when the order of the factors is changed. Example:  $8 \times 5 = 5 \times 8$ .<sup>4</sup>

<sup>4</sup> *Math at hand: A mathematics handbook* (p. 519). (1999). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		<p>Problem: Complete the multiplication sentence below, then use the commutative property to write a different multiplication sentence.</p> <p style="text-align: center;"><math>12 \times 8 = \underline{\quad}</math></p> <p>Answer: <math>96; 8 \times 12 = 96</math></p>

BIG IDEA (3): Use mathematical models to represent and understand quantitative relationships

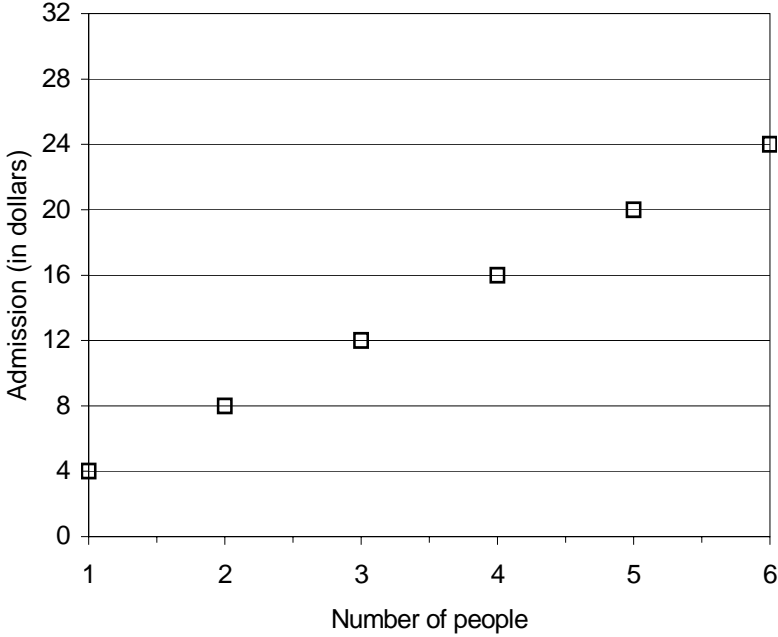
CONCEPT	EXPECTATION	EXAMPLE
A Use mathematical models	Model problem situations, using representations such as graphs, tables or number sentences	Problem: 1. The cost for admission to a fair is \$4 per person. Make a table or graph that shows how much it would cost for one to six people to attend the fair. 2. Write a number sentence to show how to find the cost of any number of tickets.

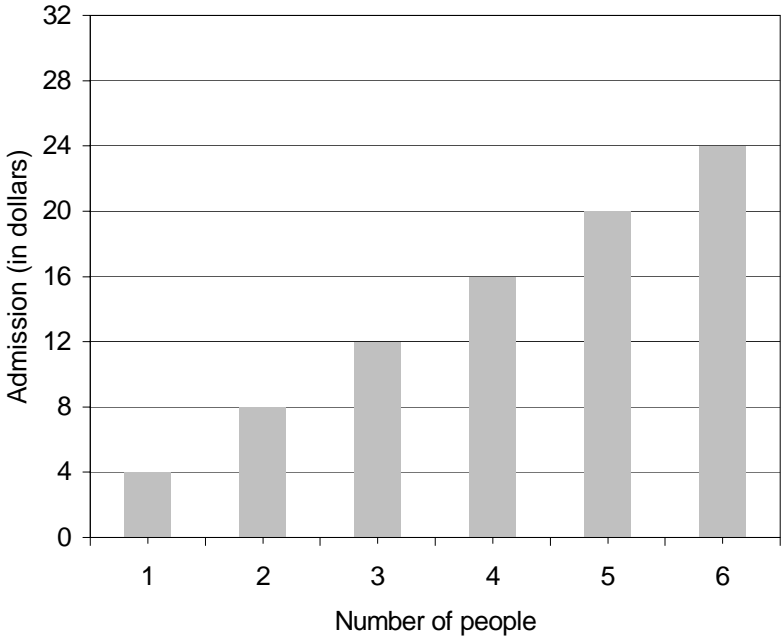
DEFINITION:

Model—to represent a mathematical situation with manipulatives (objects), pictures, numbers, or symbols.<sup>5</sup>

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<sup>5</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 95). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE														
		<p>Answer:</p> <p>1.</p> <table border="1" data-bbox="856 415 1776 513"> <tr> <td>Number of people</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>Cost</td> <td>\$4</td> <td>\$8</td> <td>\$12</td> <td>\$16</td> <td>\$20</td> <td>\$24</td> </tr> </table> <p>Numerous student solutions are possible for the graphs. Examples:</p> <p style="text-align: center;"><b>Cost of Admission Per Person</b></p> 	Number of people	1	2	3	4	5	6	Cost	\$4	\$8	\$12	\$16	\$20	\$24
Number of people	1	2	3	4	5	6										
Cost	\$4	\$8	\$12	\$16	\$20	\$24										

CONCEPT	EXPECTATION	EXAMPLE														
		<p style="text-align: center;"><b>Cost of Admission Per Person</b></p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <caption>Data for Cost of Admission Per Person</caption> <thead> <tr> <th>Number of people</th> <th>Admission (in dollars)</th> </tr> </thead> <tbody> <tr><td>1</td><td>4</td></tr> <tr><td>2</td><td>8</td></tr> <tr><td>3</td><td>12</td></tr> <tr><td>4</td><td>16</td></tr> <tr><td>5</td><td>20</td></tr> <tr><td>6</td><td>24</td></tr> </tbody> </table> <p>2. <math>\\$4 \times T</math> or <math>\\$4T</math> (<math>T</math> represents the number of tickets purchased)</p>	Number of people	Admission (in dollars)	1	4	2	8	3	12	4	16	5	20	6	24
Number of people	Admission (in dollars)															
1	4															
2	8															
3	12															
4	16															
5	20															
6	24															

CONCEPT	EXPECTATION	EXAMPLE
		<p>TEACHER NOTES:            Students in grades 3–5 can model a variety of situations, including geometric patterns, real-world situations, and scientific experiments. Sometimes they will use their model to predict the next element in a pattern. At other times, they may make a general statement about how one variable is related to another variable: for example, if a sandwich costs \$3, you can figure out how much any number of sandwiches costs by multiplying that number by 3. In modeling situations that involve real-world data, students need to know that their predictions may not always match the observed outcomes for a variety of different reasons. Students should also begin to understand that different models for the same situation can give the same results.<sup>6</sup></p>

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<sup>6</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 162–163). Reston, VA: Author.

BIG IDEA (4): Analyze change in various contexts

CONCEPT	EXPECTATION	EXAMPLE										
A Analyze change	Describe mathematical relationships in terms of constant rates of change.	<p>Problem: In the table below, how did adding one more student each time change the time spent on homework?</p> <table border="1" data-bbox="850 480 1808 623"> <tr> <td>Number of students</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>Total time spent on homework</td> <td>40 minutes</td> <td>48 minutes</td> <td>56 minutes</td> <td>64 minutes</td> </tr> </table> <p>Answer: Adding one more student increased the time spent on homework by eight minutes each time.</p> <p>TEACHER NOTES: Students should have opportunities to study situations representing different patterns of change—change that occurs at a constant rate, such as someone walking at a constant speed, and change that occurs at an increasing or decreasing rate, as in a growing plant.<sup>7</sup></p>	Number of students	5	6	7	8	Total time spent on homework	40 minutes	48 minutes	56 minutes	64 minutes
Number of students	5	6	7	8								
Total time spent on homework	40 minutes	48 minutes	56 minutes	64 minutes								

<sup>7</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

# ALGEBRAIC RELATIONSHIPS

## Grade 5

BIG IDEA (1): Understand patterns, relations and functions

CONCEPT	EXPECTATION	EXAMPLE
A Recognize and extend patterns	Make and describe generalizations about geometric and numeric patterns	<p>Problem: Find the next number in the following sequence, and explain how you know it's the next number.</p> <p>1, 4, 10, 22, ____</p> <p>Answer: The next number is 46. You take each number, double it, and add 2 to get the next number. So, <math>22 \times 2 = 44</math> plus <math>2 = 46</math>.</p>

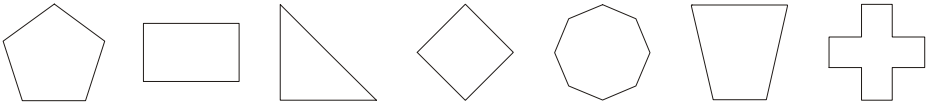
### DEFINITIONS:

generalizations—reasoning about the structure of a pattern or rule.<sup>1</sup>

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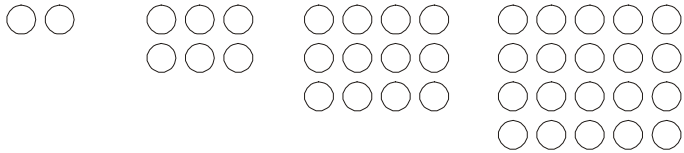
<sup>1</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 159). Reston, VA: Author.




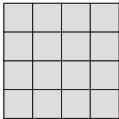
CONCEPT	EXPECTATION	EXAMPLE																				
		<p>Problem:</p> <p>John’s electronic piggy bank gives him a reading of how much money he puts in. Today he notices something strange when he puts money in. When he puts in 10 cents, the bank reports 23 cents; when he puts in 25 cents, the bank reports 53 cents; and when he puts in 32 cents, the bank reports 67 cents. Complete the Money In/Report Out table below to help you find out how much money the piggy bank would report if John deposits 45 cents. Explain how you found your answer.</p> <table border="1" data-bbox="856 565 1335 805"> <thead> <tr> <th>Money In</th> <th>Report Out</th> </tr> </thead> <tbody> <tr> <td>10 cents</td> <td>23 cents</td> </tr> <tr> <td>25 cents</td> <td>53 cents</td> </tr> <tr> <td>32 cents</td> <td>67 cents</td> </tr> <tr> <td>45 cents</td> <td></td> </tr> </tbody> </table> <p>Answer:</p> <table border="1" data-bbox="856 915 1335 1156"> <thead> <tr> <th>Money In</th> <th>Report Out</th> </tr> </thead> <tbody> <tr> <td>10 cents</td> <td>23 cents</td> </tr> <tr> <td>25 cents</td> <td>53 cents</td> </tr> <tr> <td>32 cents</td> <td>67 cents</td> </tr> <tr> <td>45 cents</td> <td>93 cents</td> </tr> </tbody> </table> <p>The report shows double the amount of money put in plus 3 cents.</p>	Money In	Report Out	10 cents	23 cents	25 cents	53 cents	32 cents	67 cents	45 cents		Money In	Report Out	10 cents	23 cents	25 cents	53 cents	32 cents	67 cents	45 cents	93 cents
Money In	Report Out																					
10 cents	23 cents																					
25 cents	53 cents																					
32 cents	67 cents																					
45 cents																						
Money In	Report Out																					
10 cents	23 cents																					
25 cents	53 cents																					
32 cents	67 cents																					
45 cents	93 cents																					

CONCEPT	EXPECTATION	EXAMPLE
		<p data-bbox="787 272 1869 381"> <b>Problem:</b>            By looking at the number of sides, John says the next shape in the following pattern could be a square. Do you think John is correct? Explain your answer.         </p> <div data-bbox="850 438 1774 544" style="text-align: center;">  </div> <p data-bbox="787 609 1869 747"> <b>Answer:</b>            Yes, it could be a square. The pattern is a shape that is not four-sided followed by a shape that is four-sided. Since the last shape in the pattern is not 4-sided, the next shape must have four sides, so a square could be the next shape.         </p> <p data-bbox="787 820 1869 1047"> <b>TEACHER NOTES:</b>            Students in grades 3–5 should investigate numerical and geometric patterns and express them mathematically in words or symbols. They should analyze structures of patterns and how they grow and change. In addition, they should organize this information systematically and use their analysis to develop generalizations about the mathematical relationships in the pattern.<sup>2</sup> </p>

<sup>2</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 159–160). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE																	
<p>B Create and analyze patterns</p>	<p>Represent and analyze patterns using words, tables and graphs</p>	<p>Problem: Make a table to show a pattern in which you start with the number 1, then add 4 to get the next number.</p> <p>Answer:</p> <table border="1" data-bbox="856 492 1526 540"> <tr> <td>1</td> <td>5</td> <td>9</td> <td>13</td> <td>17</td> <td>21</td> <td>25</td> </tr> </table> <p>Problem: What is the 4<sup>th</sup> number out in the Number In/Number Out table below? Explain how you got your answer.</p> <table border="1" data-bbox="856 764 1335 1003"> <thead> <tr> <th>Number In</th> <th>Number Out</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> </tr> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>3</td> <td>9</td> </tr> <tr> <td>4</td> <td></td> </tr> </tbody> </table> <p>Answer: The 4<sup>th</sup> number out is 16. I saw that each number in multiplied by itself gave the number out, so I took <math>4 \times 4</math> to get 16; or, I saw that the numbers out were 1 plus 3, 4 plus 5, so I added 9 plus 7 to get 16 as the 4<sup>th</sup> number out.</p>	1	5	9	13	17	21	25	Number In	Number Out	1	1	2	4	3	9	4	
1	5	9	13	17	21	25													
Number In	Number Out																		
1	1																		
2	4																		
3	9																		
4																			

CONCEPT	EXPECTATION	EXAMPLE																																
		<p>Problem:</p> <p>Frame 1      Frame 2      Frame 3      Frame 4</p> <p>  </p> <p>Use the pattern above to complete the table below. Explain how you got your answer.</p> <table border="1" data-bbox="856 695 1812 792"> <tr> <td>Frame</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>20</td> </tr> <tr> <td>Number of Shapes</td> <td>2</td> <td>6</td> <td>12</td> <td>20</td> <td></td> <td></td> <td></td> </tr> </table> <p>Answer:</p> <table border="1" data-bbox="856 979 1812 1076"> <tr> <td>Frame</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>20</td> </tr> <tr> <td>Number of Shapes</td> <td>2</td> <td>6</td> <td>12</td> <td>20</td> <td>30</td> <td>42</td> <td>420</td> </tr> </table> <p>I found the missing numbers by multiplying the frame number by itself and then adding the frame number or <math>(n \times n) + n</math>.</p>	Frame	1	2	3	4	5	6	20	Number of Shapes	2	6	12	20				Frame	1	2	3	4	5	6	20	Number of Shapes	2	6	12	20	30	42	420
Frame	1	2	3	4	5	6	20																											
Number of Shapes	2	6	12	20																														
Frame	1	2	3	4	5	6	20																											
Number of Shapes	2	6	12	20	30	42	420																											

CONCEPT	EXPECTATION	EXAMPLE
		<p data-bbox="787 272 1260 341">Problem: Describe the pattern below in words.</p> <div data-bbox="856 376 1520 553" style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p data-bbox="856 376 968 407">Frame 1</p>  </div> <div style="text-align: center;"> <p data-bbox="1016 376 1127 407">Frame 2</p>  </div> <div style="text-align: center;"> <p data-bbox="1192 376 1304 407">Frame 3</p>  </div> <div style="text-align: center;"> <p data-bbox="1402 376 1514 407">Frame 4</p>  </div> </div> <p data-bbox="787 597 1873 776">Answer: The pattern is adding the next odd number of squares each time—<math>1 + 3</math> gives the second frame of 4, and <math>4 + 5</math> gives the third frame of 9, and <math>9 + 7</math> gives the fourth frame of 16 squares; or, the pattern is the frame number multiplied by itself—<math>1 \times 1 = 1</math>, <math>2 \times 2 = 4</math>, <math>3 \times 3 = 9</math>, and <math>4 \times 4 = 16</math>.</p>

BIG IDEA (2): Represent and analyze mathematical situations and structures using algebraic symbols

CONCEPT	EXPECTATION	EXAMPLE
A Represent mathematical situations	Represent a mathematical situation as an expression or number sentence using a letter or symbol	<p>Problem: Seven buses each brought an equal number of students to school. Write an expression to show how many students rode each bus. Let <math>s</math> = the total number of students.</p> <p>Answer: <math>s \div 7</math></p> <p>Problem: Write a number sentence for five plus six minus three equals eight.</p> <p>Answer: <math>5 + 6 - 3 = 8</math></p>

DEFINITIONS:

expression—a mathematical phrase that represents a number through the combination of operation symbols, numbers, and/or symbols. Examples:  $23 \times 67$ ;  $3a$ ;  $x + y$ .<sup>3</sup>

number sentence—an equation or comparison. Examples:  $3 + 4 = 7$ ,  $8 - 2 = 6$ ,  $7 > 6$ .<sup>4</sup>

<sup>3</sup> Cavanagh, M. (2000). *Math to know* (p. 450). Wilmington, MA: Great Source Education Group, Inc.

<sup>4</sup> Cavanagh, M. (2002). *Math to learn* (p. 457). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
B Describe and use mathematical manipulation	Apply the distributive and associative properties to whole numbers	<p>Problem: Which of the following is another way to write <math>(5 + 6) \times 12</math>, using the distributive property?</p> <ol style="list-style-type: none"> <li>1. <math>5 \times 6 \times 12</math></li> <li>2. <math>(5 \times 12) + (6 \times 12)</math></li> <li>3. <math>5 + 6 + 12</math></li> </ol> <p>Answer: 2</p> <p>Problem: Use the associative or grouping property to write <math>14 \times (3 \times 5)</math> another way.</p> <p>Answer: <math>(14 \times 3) \times 5</math></p>

**DEFINITIONS:**

associative property of addition—The sum stays the same when the grouping of the addends is changed. Example:  $(5 + 4) + 6 = 5 + (4 + 6)$ .<sup>5</sup>

associative property of multiplication—The product stays the same when the grouping of the factors is changed. Example:  $(3 \times 4) \times 7 = 3 \times (4 \times 7)$ .<sup>6</sup>

distributive property—When one of the factors of a product is written as a sum, multiplying each addend before adding does not change the product. Example:  $3 \times (5 + 4) = (3 \times 5) + (3 \times 4)$ .<sup>7</sup>

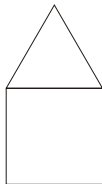
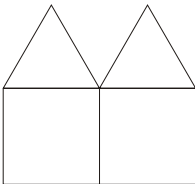
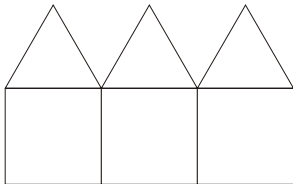
<sup>5</sup> *Math at hand: A mathematics handbook* (p. 517). (1999). Wilmington, MA: Great Source Education Group, Inc.

<sup>6</sup> *Math at hand: A mathematics handbook* (p. 517). (1999). Wilmington, MA: Great Source Education Group, Inc.

<sup>7</sup> *Math at hand: A mathematics handbook* (p. 522). (1999). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		<p>Problem: The examples below show how Allen used the distributive property to solve <math>15 \times 27</math>.</p> $15 \times (20 + 7)$ $15 \times 20 + 15 \times 7$ $300 + 150 = 405$ <p>How would Allen solve the following problem using the distributive property?</p> $12 \times 35$ <p>Answer:</p> $12 \times (30 + 5)$ $12 \times 30 + 12 \times 5$ $360 + 60 = 420$ <p>Problem: Tell what property is illustrated in each equation below.</p> <ol style="list-style-type: none"> <li>1. <math>3 + 5 + 7 = 7 + 5 + 3</math></li> <li>2. <math>(3 + 5) + 7 = 7 + (3 + 5)</math></li> <li>3. <math>(3 + 5) + 7 = 3 + (5 + 7)</math></li> <li>4. <math>3(5 + 7) = 3 \times 5 + 3 \times 7</math></li> </ol> <p>Answer:</p> <ol style="list-style-type: none"> <li>1. commutative</li> <li>2. commutative</li> <li>3. associative</li> <li>4. distributive</li> </ol>

BIG IDEA (3): Use mathematical models to represent and understand quantitative relationships

CONCEPT	EXPECTATION	EXAMPLE										
<p>A Use mathematical models</p>	<p>Model problem situations and draw conclusions, using representations such as graphs, tables or number sentence</p>	<p>Problem: Use the table and shapes below and your pattern blocks to decide what the perimeter of the 12<sup>th</sup> shape will be.</p> <table border="1" data-bbox="856 483 1335 722"> <thead> <tr> <th>Shape Number</th> <th>Perimeter</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>5</td> </tr> <tr> <td>2</td> <td>8</td> </tr> <tr> <td>3</td> <td>11</td> </tr> <tr> <td>12</td> <td></td> </tr> </tbody> </table> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 20px;"> <div style="text-align: center;">  <p>1</p> </div> <div style="text-align: center;">  <p>2</p> </div> <div style="text-align: center;">  <p>3</p> </div> </div>	Shape Number	Perimeter	1	5	2	8	3	11	12	
Shape Number	Perimeter											
1	5											
2	8											
3	11											
12												

**DEFINITION:**

Model—to represent a mathematical situation with manipulatives (objects), pictures, numbers, or symbols.<sup>8</sup>

<sup>8</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 95). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE
		<p>Answer:            The perimeter of the 12<sup>th</sup> shape will be 38. (To model this problem, some students will draw the shapes and count the sides to determine the perimeter. Others will use the pattern blocks to create the shapes and count the sides. Others may look at the table and add 3 to the previous perimeter to get the perimeter for the next number. Some may see the general pattern that the perimeter for any number can be determined by multiplying the number by 3 and adding 2.)</p> <p>TEACHER NOTES:            Students in grades 3–5 can model a variety of situations, including geometric patterns, real-world situations, and scientific experiments. Sometimes they will use their model to predict the next element in a pattern. At other times, they may make a general statement about how one variable is related to another variable: for example, if a sandwich costs \$3, you can figure out how much any number of sandwiches costs by multiplying that number by 3. In modeling situations that involve real-world data, students need to know that their predictions may not always match the observed outcomes for a variety of different reasons. Students should also begin to understand that different models for the same situation can give the same results.<sup>9</sup></p>

<sup>9</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 162–163). Reston, VA: Author.

BIG IDEA (4): Analyze change in various contexts

CONCEPT	EXPECTATION	EXAMPLE															
<p>A Analyze change</p>	<p>Identify, model and describe situations with constant or varying rates of change</p>	<p>Problem: The table below shows cell growth over several days. Is the rate of cell growth a constant or varying rate of change? Explain your thinking.</p> <table border="1" data-bbox="854 483 1570 769"> <thead> <tr> <th>Time (days)</th> <th>Number of Cells</th> <th>Change in Number of Cells</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <td>2</td> <td>2</td> <td>1</td> </tr> <tr> <td>3</td> <td>8</td> <td>7</td> </tr> <tr> <td>4</td> <td>16</td> <td>9</td> </tr> </tbody> </table> <p>Answer: It is a varying rate of change because the change in the number of cells varies from day to day.</p> <p>TEACHER NOTES: Students should have opportunities to study situations representing different patterns of change—change that occurs at a constant rate, such as someone walking at a constant speed, and change that occurs at an increasing or decreasing rate, as in a growing plant.<sup>10</sup></p>	Time (days)	Number of Cells	Change in Number of Cells	1	1	0	2	2	1	3	8	7	4	16	9
Time (days)	Number of Cells	Change in Number of Cells															
1	1	0															
2	2	1															
3	8	7															
4	16	9															

<sup>10</sup> National Council of Teachers of Mathematics (p. 163). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

# ALGEBRAIC RELATIONSHIPS

## Grade 6

BIG IDEA (1): Understand patterns, relations and functions

CONCEPT	EXPECTATION	EXAMPLE																												
B Create and analyze patterns	Represent and describe patterns with tables, graphs, pictures, symbolic rules or words	<p>Problem: Complete the table below, and write a symbolic rule for the pattern.</p> <table border="1" data-bbox="856 630 1812 727"> <tr> <td>Term number</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>Term</td> <td>3</td> <td>5</td> <td>7</td> <td></td> <td></td> <td></td> </tr> </table> <p>Answer:</p> <table border="1" data-bbox="856 841 1812 938"> <tr> <td>Term number</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>Term</td> <td>3</td> <td>5</td> <td>7</td> <td>9</td> <td>11</td> <td>13</td> </tr> </table> <p>The 4<sup>th</sup> term is 9, 5<sup>th</sup> term is 11, 6<sup>th</sup> term is 13. The symbolic rule is <math>2n + 1</math>.</p>	Term number	1	2	3	4	5	6	Term	3	5	7				Term number	1	2	3	4	5	6	Term	3	5	7	9	11	13
Term number	1	2	3	4	5	6																								
Term	3	5	7																											
Term number	1	2	3	4	5	6																								
Term	3	5	7	9	11	13																								

### DEFINITIONS:

symbolic rules—rules that use variables and numbers to describe a pattern or express a relationship.<sup>1</sup>

term number—location of a term in a sequence or pattern. For example, in the sequence 1, 3, 5, 7..., 5 is the third term number

terms—numbers, variables, products, or quotients in an expression. For example, in  $6x^2 + 5x + 3$ , there are three terms— $6x^2$ ,  $5x$ , and 3.<sup>2</sup>


<sup>1</sup> *Navigating through algebra in grades 6–8* (p. 3). (2001). Reston, VA: National Council of Teachers of Mathematics.

<sup>2</sup> Kaplan, A. (1998). *Math on call* (p. 204). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE												
		<p>Problem: A video rental store charges a \$5 membership fee and \$2 to rent a video. Make a table to show the total cost for renting one, two, three, four, and five videos. Write the symbolic rule for finding the cost for any number of videos.</p> <p>Answer:</p> <table border="1" data-bbox="856 527 1812 625"> <tbody> <tr> <td data-bbox="856 527 1234 576">Number of videos</td> <td data-bbox="1234 527 1352 576">1</td> <td data-bbox="1352 527 1470 576">2</td> <td data-bbox="1470 527 1587 576">3</td> <td data-bbox="1587 527 1705 576">4</td> <td data-bbox="1705 527 1812 576">5</td> </tr> <tr> <td data-bbox="856 576 1234 625">Term</td> <td data-bbox="1234 576 1352 625">\$7</td> <td data-bbox="1352 576 1470 625">\$9</td> <td data-bbox="1470 576 1587 625">\$11</td> <td data-bbox="1587 576 1705 625">\$13</td> <td data-bbox="1705 576 1812 625">\$15</td> </tr> </tbody> </table> <p>The symbolic rule is <math>5 + 2n</math>; <math>n</math> represents the number of videos rented.</p>	Number of videos	1	2	3	4	5	Term	\$7	\$9	\$11	\$13	\$15
Number of videos	1	2	3	4	5									
Term	\$7	\$9	\$11	\$13	\$15									

CONCEPT	EXPECTATION	EXAMPLE												
		<p>Use a table, graph, picture, or symbolic rule to represent the following sequence:</p> <p>1, 5, 9, 13, 17,...</p> <p>Answer:</p> <table border="1" data-bbox="856 492 1335 777"> <thead> <tr> <th data-bbox="856 492 1094 537">Term Number</th> <th data-bbox="1094 492 1335 537">Term Value</th> </tr> </thead> <tbody> <tr> <td data-bbox="856 537 1094 584">1</td> <td data-bbox="1094 537 1335 584">1</td> </tr> <tr> <td data-bbox="856 584 1094 631">2</td> <td data-bbox="1094 584 1335 631">5</td> </tr> <tr> <td data-bbox="856 631 1094 678">3</td> <td data-bbox="1094 631 1335 678">9</td> </tr> <tr> <td data-bbox="856 678 1094 725">4</td> <td data-bbox="1094 678 1335 725">13</td> </tr> <tr> <td data-bbox="856 725 1094 773">5</td> <td data-bbox="1094 725 1335 773">17</td> </tr> </tbody> </table>	Term Number	Term Value	1	1	2	5	3	9	4	13	5	17
Term Number	Term Value													
1	1													
2	5													
3	9													
4	13													
5	17													

CONCEPT	EXPECTATION	EXAMPLE
		<p style="text-align: center;"><b>Sequence</b></p> <p style="text-align: center;">Term value</p> <p style="text-align: center;">Term number</p> <div style="text-align: center; margin-top: 20px;"> </div> <p style="text-align: center; margin-top: 20px;">The symbolic rule is <math>4n - 3</math>.</p>

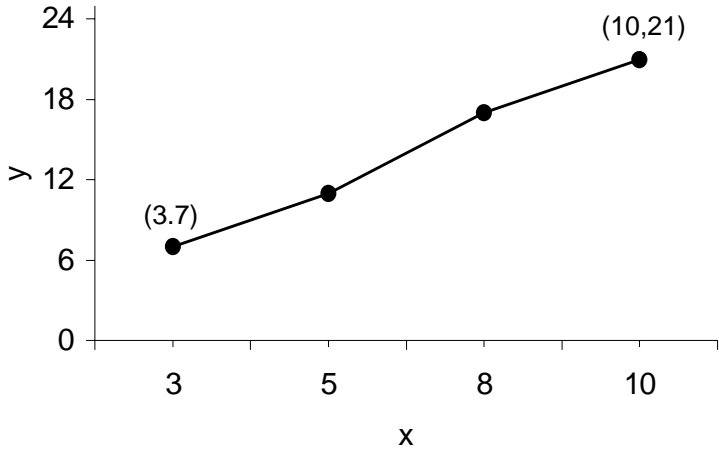
CONCEPT	EXPECTATION	EXAMPLE														
C Classify objects and representations	Compare various forms of representations to identify a pattern	<p>Problem: A sequence can be formed by arranging toothpicks to make rectangles as shown:</p> <p style="text-align: center;">Frame 1      Frame 2      Frame 3</p> <p style="text-align: center;">  </p> <p>Make a table, and record the first six term numbers and the number of toothpicks needed for each term number. Write a general rule describing the relationship between the term number (<math>n</math>) and the number of toothpicks. Use the general rule to find the number of toothpicks needed for the 500<sup>th</sup> rectangle.</p> <p>Answer:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td style="padding: 5px;">Frame number</td> <td style="padding: 5px; text-align: center;">1</td> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px; text-align: center;">3</td> <td style="padding: 5px; text-align: center;">4</td> <td style="padding: 5px; text-align: center;">5</td> <td style="padding: 5px; text-align: center;">6</td> </tr> <tr> <td style="padding: 5px;">Number of toothpicks</td> <td style="padding: 5px; text-align: center;">4</td> <td style="padding: 5px; text-align: center;">7</td> <td style="padding: 5px; text-align: center;">10</td> <td style="padding: 5px; text-align: center;">13</td> <td style="padding: 5px; text-align: center;">16</td> <td style="padding: 5px; text-align: center;">19</td> </tr> </tbody> </table> <p>The general rule for describing the relationship between the term number and the number of toothpicks is <math>3n + 1</math>. The 500th rectangle would require <math>(500 \times 3) + 1</math>, or 1501 toothpicks.</p>	Frame number	1	2	3	4	5	6	Number of toothpicks	4	7	10	13	16	19
Frame number	1	2	3	4	5	6										
Number of toothpicks	4	7	10	13	16	19										

**DEFINITION:**

representations—physical objects, drawings, charts, graphs, and symbols that help students communicate their thinking.<sup>3</sup>

<sup>3</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 280). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE														
		<p>Problem: Find the 625<sup>th</sup> term in the sequence represented below. Explain how the total is related to the term number.</p> <table border="1" data-bbox="858 418 1814 513"> <tbody> <tr> <td data-bbox="858 418 1283 464">Term number</td> <td data-bbox="1283 418 1371 464">1</td> <td data-bbox="1371 418 1459 464">2</td> <td data-bbox="1459 418 1547 464">3</td> <td data-bbox="1547 418 1635 464">4</td> <td data-bbox="1635 418 1724 464">5</td> <td data-bbox="1724 418 1814 464">625</td> </tr> <tr> <td data-bbox="858 464 1283 513">Total</td> <td data-bbox="1283 464 1371 513">3</td> <td data-bbox="1371 464 1459 513">5</td> <td data-bbox="1459 464 1547 513">7</td> <td data-bbox="1547 464 1635 513">9</td> <td data-bbox="1635 464 1724 513">11</td> <td data-bbox="1724 464 1814 513"></td> </tr> </tbody> </table> <p>Answer: The 625<sup>th</sup> term is <math>(2 \times 625) + 1</math>, or 1,251. The total is one more than twice the term number, i.e., <math>2n + 1</math>.</p>	Term number	1	2	3	4	5	625	Total	3	5	7	9	11	
Term number	1	2	3	4	5	625										
Total	3	5	7	9	11											

CONCEPT	EXPECTATION	EXAMPLE										
		<p data-bbox="787 272 1680 341">Problem: Describe how the following graphics can represent the same pattern.</p> <table border="1" data-bbox="856 378 1146 618"> <thead> <tr> <th>Input</th> <th>Output</th> </tr> </thead> <tbody> <tr> <td>3</td> <td>7</td> </tr> <tr> <td>5</td> <td>11</td> </tr> <tr> <td>8</td> <td>17</td> </tr> <tr> <td>10</td> <td>21</td> </tr> </tbody> </table>  <p data-bbox="787 1182 1879 1365">Answer: <i>(Answers may vary.)</i> The input in the table represents the <math>x</math> term of the graph, and the output represents the <math>y</math> term of the graph. The table and graph show a linear function because there is a constant rate of change of 6 from one term to the next, a change in the <math>x</math> value causes a proportional change in the <math>y</math> value in both the table and the graph.</p>	Input	Output	3	7	5	11	8	17	10	21
Input	Output											
3	7											
5	11											
8	17											
10	21											

CONCEPT	EXPECTATION	EXAMPLE																
		<p>Problem: Match the sequence with the appropriate description or rule.</p> <table border="1" data-bbox="858 380 1717 714"> <thead> <tr> <th colspan="2" data-bbox="858 380 1289 427">Sequences</th> <th colspan="2" data-bbox="1289 380 1717 427">Rules</th> </tr> </thead> <tbody> <tr> <td data-bbox="858 427 953 524">1.</td> <td data-bbox="953 427 1289 524">10, 12, 14, 16, 18</td> <td data-bbox="1289 427 1383 524">A.</td> <td data-bbox="1383 427 1717 524">Double the previous term.</td> </tr> <tr> <td data-bbox="858 524 953 621">2.</td> <td data-bbox="953 524 1289 621">1, 2, 4, 8, 16</td> <td data-bbox="1289 524 1383 621">B.</td> <td data-bbox="1383 524 1717 621">Add 2 to the previous term.</td> </tr> <tr> <td data-bbox="858 621 953 714">3.</td> <td data-bbox="953 621 1289 714">14, 11, 8, 5, 2</td> <td data-bbox="1289 621 1383 714">C.</td> <td data-bbox="1383 621 1717 714">Subtract 3 from the previous term.</td> </tr> </tbody> </table> <p>Answer: 1. Rule B 2. Rule A 3. Rule C</p>	Sequences		Rules		1.	10, 12, 14, 16, 18	A.	Double the previous term.	2.	1, 2, 4, 8, 16	B.	Add 2 to the previous term.	3.	14, 11, 8, 5, 2	C.	Subtract 3 from the previous term.
Sequences		Rules																
1.	10, 12, 14, 16, 18	A.	Double the previous term.															
2.	1, 2, 4, 8, 16	B.	Add 2 to the previous term.															
3.	14, 11, 8, 5, 2	C.	Subtract 3 from the previous term.															

CONCEPT	EXPECTATION	EXAMPLE												
D Identify and compare functions	Identify functions as linear or nonlinear from a table or graph	<p>Problem: Suppose you put \$100 in the bank and your money doubles every seven years as shown below:</p> <table border="1" data-bbox="856 418 1814 513"> <tr> <td>Years</td> <td>0</td> <td>7</td> <td>14</td> <td>21</td> <td>28</td> </tr> <tr> <td>Amount in bank</td> <td>\$100</td> <td>\$200</td> <td>\$400</td> <td>\$800</td> <td>\$1,600</td> </tr> </table> <p>The data of your savings is shown in the graph below. Discuss whether the rate of change is constant from one point to the next and whether the graph is linear or non-linear and why.</p>	Years	0	7	14	21	28	Amount in bank	\$100	\$200	\$400	\$800	\$1,600
Years	0	7	14	21	28									
Amount in bank	\$100	\$200	\$400	\$800	\$1,600									

**DEFINITIONS:**

functions—relations in which every value of  $x$  has a unique value of  $y$ .<sup>4</sup>

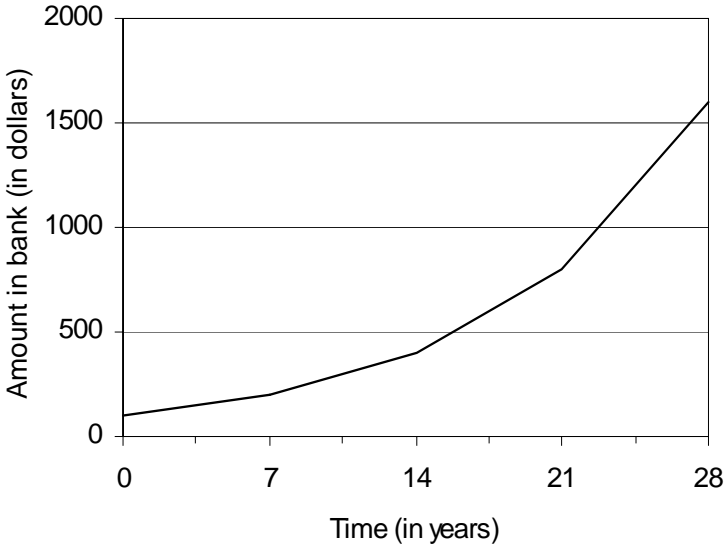
linear (function) equation—“an equation whose graph in a coordinate grid is a straight line.”<sup>5</sup>

nonlinear (function) equation—a function whose graph in a coordinate grid is not a straight line.<sup>6</sup>

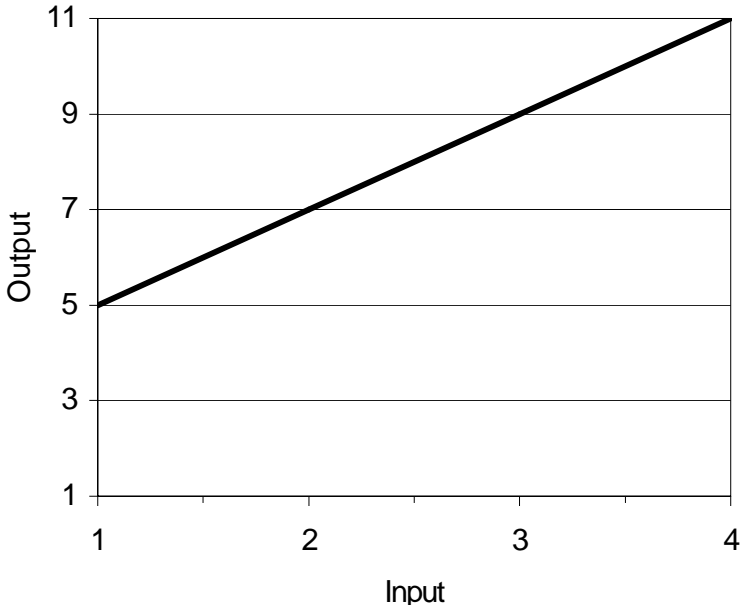
<sup>4</sup> Kaplan, A. (1998). *Math on call* (p. 583). Wilmington, MA: Great Source Education Group, Inc.

<sup>5</sup> Kaplan, A. (1998). *Math on call* (p. 583). Wilmington, MA: Great Source Education Group, Inc.

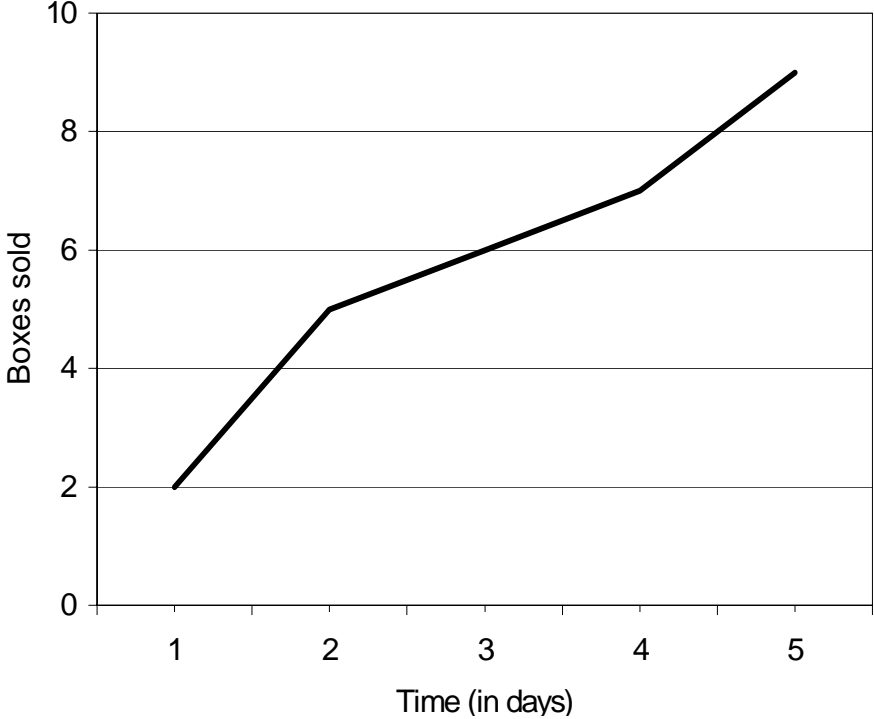
<sup>6</sup> *Algebra to go* (p. 459). (1998). Wilmington, MA: Great Source Education Group, Inc..

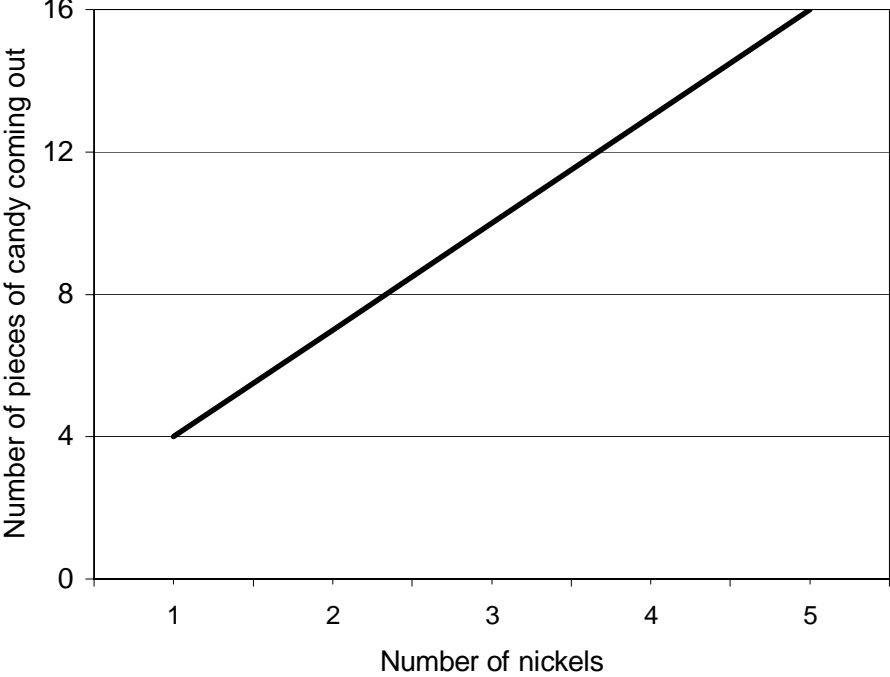
CONCEPT	EXPECTATION	EXAMPLE
		<p style="text-align: center;"><b>Money in the Bank</b></p>  <p style="text-align: center;">Time (in years)</p> <p>Amount in bank (in dollars)</p> <p>Answer: The rate of change is not constant. The first seven years, the amount increases \$100; the second seven years, it increases \$200; and the third seven years, the amount increases \$400. Since the rate of change, or slope, is not constant, the graph is non-linear.</p>

CONCEPT	EXPECTATION	EXAMPLE										
		<p>Problem: Given the following table of numbers, explain whether or not the pattern is linear.</p> <table border="1" data-bbox="863 370 1150 609"> <thead> <tr> <th data-bbox="863 370 1005 418">Input</th> <th data-bbox="1005 370 1150 418">Output</th> </tr> </thead> <tbody> <tr> <td data-bbox="863 418 1005 467">1</td> <td data-bbox="1005 418 1150 467">5</td> </tr> <tr> <td data-bbox="863 467 1005 516">2</td> <td data-bbox="1005 467 1150 516">7</td> </tr> <tr> <td data-bbox="863 516 1005 565">3</td> <td data-bbox="1005 516 1150 565">9</td> </tr> <tr> <td data-bbox="863 565 1005 609">4</td> <td data-bbox="1005 565 1150 609">11</td> </tr> </tbody> </table>	Input	Output	1	5	2	7	3	9	4	11
Input	Output											
1	5											
2	7											
3	9											
4	11											

CONCEPT	EXPECTATION	EXAMPLE										
		<p>Answer:            The pattern is linear, since the rate of change is constant. The output increases by 2 each time, while the input increases by 1.</p> <p><i>Some students may draw a graph to justify their answer.</i></p> <p style="text-align: center;"><b>Change in Output</b></p>  <table border="1" data-bbox="871 560 1606 1161"> <caption>Data points from the graph</caption> <thead> <tr> <th>Input</th> <th>Output</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>5</td> </tr> <tr> <td>2</td> <td>7</td> </tr> <tr> <td>3</td> <td>9</td> </tr> <tr> <td>4</td> <td>11</td> </tr> </tbody> </table>	Input	Output	1	5	2	7	3	9	4	11
Input	Output											
1	5											
2	7											
3	9											
4	11											

CONCEPT	EXPECTATION	EXAMPLE												
		<p>Problem: Identify the non-linear graph below and explain why it is a non-linear graph.</p> <p>A.</p> <p style="text-align: center;"><b>Change in Distance Traveled by an Airplane</b></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <caption>Data points for the graph</caption> <thead> <tr> <th>Time (in hours)</th> <th>Distance traveled (in miles)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>500</td> </tr> <tr> <td>2</td> <td>1000</td> </tr> <tr> <td>3</td> <td>1500</td> </tr> <tr> <td>4</td> <td>2000</td> </tr> <tr> <td>5</td> <td>2500</td> </tr> </tbody> </table>	Time (in hours)	Distance traveled (in miles)	1	500	2	1000	3	1500	4	2000	5	2500
Time (in hours)	Distance traveled (in miles)													
1	500													
2	1000													
3	1500													
4	2000													
5	2500													

CONCEPT	EXPECTATION	EXAMPLE												
		<p data-bbox="856 272 888 297">B.</p> <p data-bbox="1058 306 1650 337"><b>Change in Number of Boxes of Cookies Sold</b></p>  <table border="1" data-bbox="940 375 1808 1089"> <caption>Data points for 'Change in Number of Boxes of Cookies Sold'</caption> <thead> <tr> <th>Time (in days)</th> <th>Boxes sold</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> </tr> <tr> <td>2</td> <td>5</td> </tr> <tr> <td>3</td> <td>6</td> </tr> <tr> <td>4</td> <td>7</td> </tr> <tr> <td>5</td> <td>9</td> </tr> </tbody> </table>	Time (in days)	Boxes sold	1	2	2	5	3	6	4	7	5	9
Time (in days)	Boxes sold													
1	2													
2	5													
3	6													
4	7													
5	9													

CONCEPT	EXPECTATION	EXAMPLE
		<p data-bbox="856 272 890 298">C.</p> <p data-bbox="1184 318 1556 344" style="text-align: center;"><b>Number of Candy Per Nickel</b></p>  <p data-bbox="793 1107 905 1133">Answer:</p> <p data-bbox="793 1143 1856 1211">Graph B is non-linear because the rate of change is not constant, and the graph is not a straight line.</p>

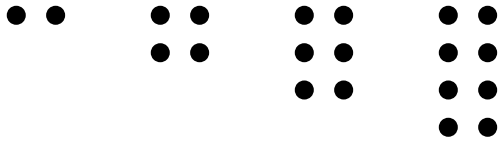
BIG IDEA (2): Represent and analyze mathematical situations and structures using algebraic symbols

CONCEPT	EXPECTATION	EXAMPLE
A Represent mathematical situations	Use variables to represent unknown quantities in expressions	<p>Problem: Which picture below represents the expression <math>2h + 3s</math>? Explain why.</p> <p>A. MMMmmm B. MMmmm C. MMmm D. Mm</p> <p>Answer: B <math>h</math> is for heart, and <math>s</math> is for star, and the expression is 2 hearts (<math>h</math>) and 3 stars (<math>s</math>), or <math>2h + 3s</math>.</p>

CONCEPT	EXPECTATION	EXAMPLE																																
		<p>Problem:</p> <p>In the table shown below,</p> <ol style="list-style-type: none"> <li>complete the next three terms;</li> <li>explain how the terms are related to the term numbers (<math>n</math>);</li> <li>write a general rule—term = _____;</li> <li>show how you use your rule to find the 30<sup>th</sup> term.</li> </ol> <table border="1" data-bbox="852 529 1808 625"> <tr> <td>Term number</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> </tr> <tr> <td>Term</td> <td>12</td> <td>24</td> <td>36</td> <td>48</td> <td></td> <td></td> <td></td> </tr> </table> <p>Answer:</p> <ol style="list-style-type: none"> <li> <table border="1" data-bbox="852 820 1808 915"> <tr> <td>Term number</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> </tr> <tr> <td>Term</td> <td>12</td> <td>24</td> <td>36</td> <td>48</td> <td>60</td> <td>72</td> <td>84</td> </tr> </table> <ol style="list-style-type: none"> <li>The term is the term number times 12.</li> <li>Term = term number <math>\times</math> 12, or <math>n \times 12</math>.</li> <li>The 30<sup>th</sup> term is <math>30 \times 12</math>, or 360.</li> </ol> </li> </ol>	Term number	1	2	3	4	5	6	7	Term	12	24	36	48				Term number	1	2	3	4	5	6	7	Term	12	24	36	48	60	72	84
Term number	1	2	3	4	5	6	7																											
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Term	12	24	36	48	60	72	84																											

CONCEPT	EXPECTATION	EXAMPLE																																
		<p>Problem:</p> <p>In the table shown below,</p> <ol style="list-style-type: none"> <li>complete the missing terms;</li> <li>write a symbolic rule to find any missing term.</li> </ol> <table border="1" data-bbox="852 456 1808 550"> <tr> <td>Term number</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>25</td> <td>200</td> </tr> <tr> <td>Term</td> <td>8</td> <td>15</td> <td>22</td> <td>29</td> <td></td> <td></td> <td></td> </tr> </table> <p>Answer:</p> <ol style="list-style-type: none"> <li> <table border="1" data-bbox="852 745 1808 839"> <tr> <td>Term number</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>25</td> <td>200</td> </tr> <tr> <td>Term</td> <td>8</td> <td>15</td> <td>22</td> <td>29</td> <td>36</td> <td>176</td> <td>1,401</td> </tr> </table> </li> <li>The symbolic rule to find any missing term is <math>7n + 1</math>.</li> </ol>	Term number	1	2	3	4	5	25	200	Term	8	15	22	29				Term number	1	2	3	4	5	25	200	Term	8	15	22	29	36	176	1,401
Term number	1	2	3	4	5	25	200																											
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Term number	1	2	3	4	5	25	200																											
Term	8	15	22	29	36	176	1,401																											

CONCEPT	EXPECTATION	EXAMPLE
		<p>Problem: Which of the following expressions represents three more than twice a number?</p> <p>A. <math>3n + 2</math> B. <math>5n</math> C. <math>2n + 3</math> D. <math>3n - 2</math></p> <p>Answer: C</p>

CONCEPT	EXPECTATION	EXAMPLE
		<p>Problem: Use a variable to write an expression that determines the number of dots, no matter what the term number is.</p>  <p>Answer: The term is equal to the term number times two, or <math>n \times 2</math> (<math>n = 1, 2, 3, 4, \dots</math>).</p> <p>Problem: Use a variable to write an expression for determining any number (term) in the sequence.</p> <p>3, 7, 11, 15...</p> <p>Answer: <math>4n - 1</math></p>

CONCEPT	EXPECTATION	EXAMPLE
B Describe and use mathematical manipulation	Recognize equivalent forms for simple algebraic expressions (associative, distributive properties)	<p>Problem: Does changing the grouping in the following expressions affect the sum? Explain why or why not.</p> <p style="text-align: center;"><math>(5 + 9) + 12</math>      <math>5 + (9 + 12)</math></p> <p>Answer: No. Addition is associative, and either side equals 26.</p>

**DEFINITIONS:**

associative property of addition—The sum stays the same when the grouping of the addends is changed.

Example:  $(22 + 13) + 12 = 22 + (13 + 12)$ .<sup>7</sup>

associative property of multiplication—The product stays the same when the grouping of the factors is changed.

Example:  $(8 \times 7) \times 13 = 8 \times (7 \times 13)$ .<sup>8</sup>

distributive property—When one of the factors of a product is written as a sum, multiplying each addend before adding does not change the product. Example:  $7 \times (11 + 13) = (7 \times 11) + (7 \times 13)$ .<sup>9</sup>

<sup>7</sup> *Math at hand: A mathematics handbook* (p. 517). (1999). Wilmington, MA: Great Source Education Group, Inc.

<sup>8</sup> *Math at hand: A mathematics handbook* (p. 517). (1999). Wilmington, MA: Great Source Education Group, Inc.

<sup>9</sup> *Math at hand: A mathematics handbook* (p. 522). (1999). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		<p>Problem: Does changing the grouping in the following expression affect the product? Explain why or why not.</p> $(5 \times b) \times 7 \qquad 5 \times (b \times 7)$ <p>Answer: No. Multiplication is associative, and either side equals <math>35b</math>.</p> <p>Problem: Does <math>(7 + 8) \times 3</math> equal the same as <math>7 + (8 \times 3)</math>? Show your work to support your answer.</p> <p>Answer: No. You have to do what's in parentheses in each problem first. In the first problem, you add <math>7 + 8</math> to get 15, and 15 times 3 is 45. In the second problem, you multiply <math>8 \times 3</math> first to get 24, and add 7 for a total of 31. 45 is not equal to 31.</p>

CONCEPT	EXPECTATION	EXAMPLE
		<p>Problem: Show why the following equation is true:</p> $5(3 + 4) = (5 \times 3) + (5 \times 4)$ <p>then show how to expand <math>5(x + 4)</math>.</p> <p>Answer:</p> $5(7) = (15 + 20)$ $35 = 35$ $5(x + 4) = 5x + 20$ <p>Problem: Tell what property is illustrated in each equation below.</p> <ol style="list-style-type: none"> <li>1. <math>a + b + c = c + b + a</math></li> <li>2. <math>(a + b) + c = c + (a + b)</math></li> <li>3. <math>(a + b) + c = a + (b + c)</math></li> <li>4. <math>a(b + a) = ab + ac</math></li> </ol> <p>Answer:</p> <ol style="list-style-type: none"> <li>1. commutative</li> <li>2. commutative</li> <li>3. associative</li> <li>4. distributive</li> </ol>

CONCEPT	EXPECTATION	EXAMPLE								
		<p>Problem: Which of the following are equivalent to <math>n + 9</math>?</p> <table border="1" data-bbox="856 380 1115 571"> <tbody> <tr> <td data-bbox="856 380 926 427">1.</td> <td data-bbox="926 380 1115 427"><math>9 + n</math></td> </tr> <tr> <td data-bbox="856 427 926 474">2.</td> <td data-bbox="926 427 1115 474"><math>9n</math></td> </tr> <tr> <td data-bbox="856 474 926 521">3.</td> <td data-bbox="926 474 1115 521"><math>3 + n + 6</math></td> </tr> <tr> <td data-bbox="856 521 926 571">4.</td> <td data-bbox="926 521 1115 571"><math>n - 9</math></td> </tr> </tbody> </table> <p>A. 1 only            B. 2 only            C. 3 only            D. 4 only            E. 1 and 3 only            F. 1, 2, and 3 only</p> <p>Answer: E</p>	1.	$9 + n$	2.	$9n$	3.	$3 + n + 6$	4.	$n - 9$
1.	$9 + n$									
2.	$9n$									
3.	$3 + n + 6$									
4.	$n - 9$									

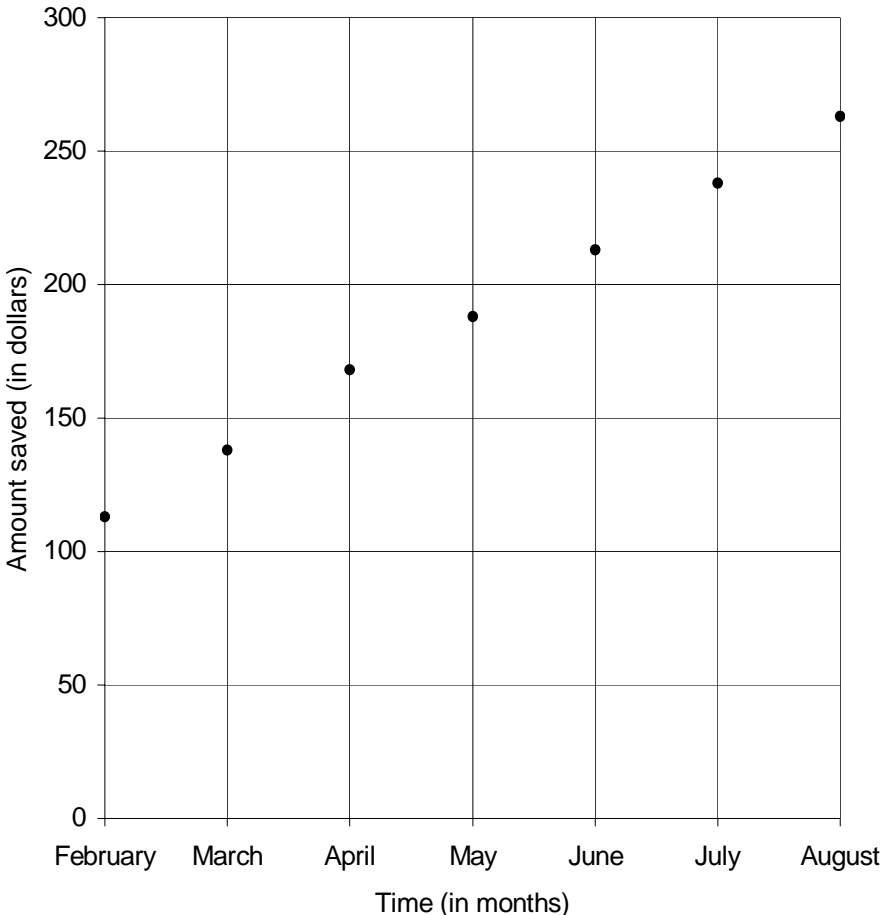
BIG IDEA (3): Use mathematical models to represent and understand quantitative relationships

CONCEPT	EXPECTATION	EXAMPLE																								
A Use mathematical models	Model and solve problems, using multiple representations such as graphs, tables, expressions and equations	<p>Problem:</p> <p>At the end of February, Ben began to save for a \$240 mountain bike. At the time, he had \$113 in savings. His savings increased to \$138 in March and \$163 in April. Complete the table below to determine when Ben will have enough if he continues to save at the same rate:</p> <table border="1" data-bbox="856 558 1457 654"> <tr> <td>Month</td> <td>Feb</td> <td>Mar</td> <td>April</td> </tr> <tr> <td>Amount saved</td> <td>\$113</td> <td>\$138</td> <td>\$163</td> </tr> </table> <p>Draw a coordinate graph reflecting the information in the table. Describe any patterns you find in the table or graph.</p> <p>Answer:</p> <table border="1" data-bbox="856 878 1814 974"> <tr> <td>Month</td> <td>Feb</td> <td>Mar</td> <td>April</td> <td>May</td> <td>June</td> <td>July</td> <td>Aug</td> </tr> <tr> <td>Amount saved</td> <td>\$113</td> <td>\$138</td> <td>\$163</td> <td>\$188</td> <td>\$213</td> <td>\$238</td> <td>\$263</td> </tr> </table>	Month	Feb	Mar	April	Amount saved	\$113	\$138	\$163	Month	Feb	Mar	April	May	June	July	Aug	Amount saved	\$113	\$138	\$163	\$188	\$213	\$238	\$263
Month	Feb	Mar	April																							
Amount saved	\$113	\$138	\$163																							
Month	Feb	Mar	April	May	June	July	Aug																			
Amount saved	\$113	\$138	\$163	\$188	\$213	\$238	\$263																			

DEFINITION:

Model—to represent a mathematical situation with manipulatives (objects), pictures, numbers, or symbols.<sup>10</sup>

<sup>10</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 95). Reston, VA: Author.

CONCEPT	EXPECTATION	EXAMPLE																
		<p style="text-align: center;"><b>Change in Savings</b></p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <caption>Data for Change in Savings</caption> <thead> <tr> <th>Time (in months)</th> <th>Amount saved (in dollars)</th> </tr> </thead> <tbody> <tr> <td>February</td> <td>110</td> </tr> <tr> <td>March</td> <td>140</td> </tr> <tr> <td>April</td> <td>170</td> </tr> <tr> <td>May</td> <td>190</td> </tr> <tr> <td>June</td> <td>215</td> </tr> <tr> <td>July</td> <td>240</td> </tr> <tr> <td>August</td> <td>265</td> </tr> </tbody> </table> <p>Ben will have enough money for his bike in August. The pattern is that \$25 is added on each month.</p>	Time (in months)	Amount saved (in dollars)	February	110	March	140	April	170	May	190	June	215	July	240	August	265
Time (in months)	Amount saved (in dollars)																	
February	110																	
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CONCEPT	EXPECTATION	EXAMPLE																		
		<p>Problem:            'Talk a Lot' phone company charges \$15 a month for service and \$3 for every hour of long-distance calls. Mary talks long distance for two hours one month, and Tom talks long distance for four hours the same month. Using an equation or table, determine the charge that month for both Mary and Tom. Describe or explain how to determine the price for any number of hours.</p> <p>Answer:            Each person's charges can be determined by using <math>\\$15 + \\$3n</math>. The following equations show the cost for each.            Mary—<math>15 + 3(2) = 21</math>            Tom—<math>15 + 3(4) = 27</math></p> <p>The cost for any number of hours (<math>n</math>) can be found by using <math>\\$15 + 3n</math>, where <math>n</math> is the number of hours talked.</p> <table border="1" data-bbox="856 862 1801 1008"> <thead> <tr> <th></th> <th>Initial</th> <th>Hour 1</th> <th>Hour 2</th> <th>Hour 3</th> <th>Hour 4</th> </tr> </thead> <tbody> <tr> <td>Mary</td> <td>\$15</td> <td>\$18</td> <td>\$21</td> <td></td> <td></td> </tr> <tr> <td>Tom</td> <td>\$15</td> <td>\$18</td> <td>\$21</td> <td>\$24</td> <td>\$27</td> </tr> </tbody> </table> <p>TEACHER NOTES:            Students in grades 3–5 can model a variety of situations, including geometric patterns, real-world situations, and scientific experiments. Sometimes they will use their model to predict the next element in a pattern. At other times, they may make a general statement about how one variable is related to another variable: for</p>		Initial	Hour 1	Hour 2	Hour 3	Hour 4	Mary	\$15	\$18	\$21			Tom	\$15	\$18	\$21	\$24	\$27
	Initial	Hour 1	Hour 2	Hour 3	Hour 4															
Mary	\$15	\$18	\$21																	
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CONCEPT	EXPECTATION	EXAMPLE
		<p>example, if a sandwich costs \$3, you can figure out how much any number of sandwiches costs by multiplying that number by 3. In modeling situations that involve real-world data, students need to know that their predictions may not always match the observed outcomes for a variety of different reasons. Students should also begin to understand that different models for the same situation can give the same results.<sup>11</sup></p>

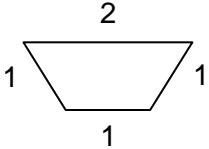


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<sup>11</sup> National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (pp. 162–163). Reston, VA: Author.

BIG IDEA (4): Analyze change in various contexts

CONCEPT	EXPECTATION	EXAMPLE																										
A Analyze change	Compare situations with constant or varying rates of change	<p>Problem: Consider a rectangle with a fixed area of 12 square units. Make a table showing the widths for all possible whole-number lengths of the rectangle up to a length of 12. Would a graph of the relationship between L (length) and W (width) be a straight line? Why or why not?</p> <p>Answer:</p> <table border="1" data-bbox="856 631 1812 727"> <tbody> <tr> <td>Length</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> <td>11</td> <td>12</td> </tr> <tr> <td>Width</td> <td>12</td> <td>6</td> <td>4</td> <td>3</td> <td>2.4</td> <td>2</td> <td>1.7</td> <td>1.5</td> <td>1.3</td> <td>1.2</td> <td>1.1</td> <td>1</td> </tr> </tbody> </table> <p>The graph would not be a straight line. As the length increases by a constant rate of 1, the width decreases, but not at a constant rate.</p>	Length	1	2	3	4	5	6	7	8	9	10	11	12	Width	12	6	4	3	2.4	2	1.7	1.5	1.3	1.2	1.1	1
Length	1	2	3	4	5	6	7	8	9	10	11	12																
Width	12	6	4	3	2.4	2	1.7	1.5	1.3	1.2	1.1	1																

CONCEPT	EXPECTATION	EXAMPLE																																										
		<p>Problem:</p> <p>1. If you rent 10 videos, would it be cheaper to pay a \$15 membership fee and \$2 to rent a video, or no membership fee and \$3 to rent a video? 2. For what number of videos would the price be the same? Show how you got your answer.</p> <p>Answer:</p> <p>1. <math>15 + 2(10) = 35</math>  <math>3(10) = 30</math>  The second method is cheaper.</p> <p>2. 15 videos  <math>15 + 2(15) = 45</math> and <math>3(15) = 45</math></p> <p><i>Some students may draw a table to show their answer. Example:</i></p> <table border="1" data-bbox="856 862 1810 1006"> <tr> <td colspan="7">\$15 membership fee and \$2 to rent a video</td> </tr> <tr> <td>Number of videos</td> <td>0</td> <td>1</td> <td>5</td> <td>10</td> <td>15</td> <td>20</td> </tr> <tr> <td>Cost for videos</td> <td>\$15</td> <td>\$17</td> <td>\$25</td> <td>\$35</td> <td>\$45</td> <td>\$55</td> </tr> </table> <table border="1" data-bbox="856 1045 1810 1190"> <tr> <td colspan="7">No membership fee and \$3 to rent a video</td> </tr> <tr> <td>Number of videos</td> <td>0</td> <td>1</td> <td>5</td> <td>10</td> <td>15</td> <td>20</td> </tr> <tr> <td>Cost for videos</td> <td>\$0</td> <td>\$3</td> <td>\$15</td> <td>\$30</td> <td>\$45</td> <td>\$60</td> </tr> </table>	\$15 membership fee and \$2 to rent a video							Number of videos	0	1	5	10	15	20	Cost for videos	\$15	\$17	\$25	\$35	\$45	\$55	No membership fee and \$3 to rent a video							Number of videos	0	1	5	10	15	20	Cost for videos	\$0	\$3	\$15	\$30	\$45	\$60
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CONCEPT	EXPECTATION	EXAMPLE																																
		<p>Problem:</p> <p>In the following pattern,</p> <ol style="list-style-type: none"> <li>find the perimeters of the next four sets of quadrilaterals, and explain how you got your answer;</li> <li>find the perimeter of the 100<sup>th</sup> quadrilateral.</li> </ol> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>Quadrilateral 1 Perimeter = 5</p> </div> <div style="text-align: center;">  <p>Quadrilateral 2 Perimeter = 8</p> </div> <div style="text-align: center;">  <p>Quadrilateral 3 Perimeter = 11</p> </div> </div> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 15%;">Quadrilateral</td> <td style="width: 5%;">1</td> <td style="width: 5%;">2</td> <td style="width: 5%;">3</td> <td style="width: 5%;">4</td> <td style="width: 5%;">5</td> <td style="width: 5%;">6</td> <td style="width: 5%;">7</td> </tr> <tr> <td>Perimeter</td> <td>5</td> <td>8</td> <td>11</td> <td></td> <td></td> <td></td> <td></td> </tr> </table> <p>Answer:</p> <ol style="list-style-type: none"> <li>The perimeter increases by three each time.</li> </ol> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 15%;">Quadrilateral</td> <td style="width: 5%;">1</td> <td style="width: 5%;">2</td> <td style="width: 5%;">3</td> <td style="width: 5%;">4</td> <td style="width: 5%;">5</td> <td style="width: 5%;">6</td> <td style="width: 5%;">7</td> </tr> <tr> <td>Perimeter</td> <td>5</td> <td>8</td> <td>11</td> <td>14</td> <td>17</td> <td>20</td> <td>23</td> </tr> </table> <ol style="list-style-type: none"> <li><math>3x + 2</math> will give the perimeter of any quadrilateral. So, <math>3(100) + 2 = 302</math>.</li> </ol>	Quadrilateral	1	2	3	4	5	6	7	Perimeter	5	8	11					Quadrilateral	1	2	3	4	5	6	7	Perimeter	5	8	11	14	17	20	23
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